

SUSY Q-balls as a dark matter candidate

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- What are Q-balls?
- Supersymmetric Q-balls
- Baryon number violation and Q-balls
- SUSY Q-balls as dark matter candidates
- Cosmological production
- Matter-antimatter conversion
- Experimental signatures
- Astrophysical constraints
- Conclusions

Simplest example: one scalar field with unbroken U(1) global symmetry:

$$L = \partial_\mu \phi^* \partial^\mu \phi - U(\phi^* \phi)$$

Conserved charge:

$$Q = i \int d^3x [\phi^* (\partial_0 \phi) - (\partial_0 \phi^*) \phi]$$

Question:

What is the minimum energy configuration with the charge Q?

What are Q-balls?

Answers:

- Collection of Q well separated scalar particles, $E = mQ$
- If the particles with the same charge are attracted to each other:
bound state of Q scalar particles \equiv **Q-ball**, $E = (m - \Delta m)Q$

Mathematics: find a minimum of energy functional

$$E = \int d^3x \left[|\partial_0 \phi|^2 + |\vec{\nabla} \phi|^2 + U(\phi^* \phi) \right]$$

keeping the charge

$$Q = i \int d^3x [\phi^* (\partial_0 \phi) - (\partial_0 \phi^*) \phi]$$

fixed.

What are Q-balls?

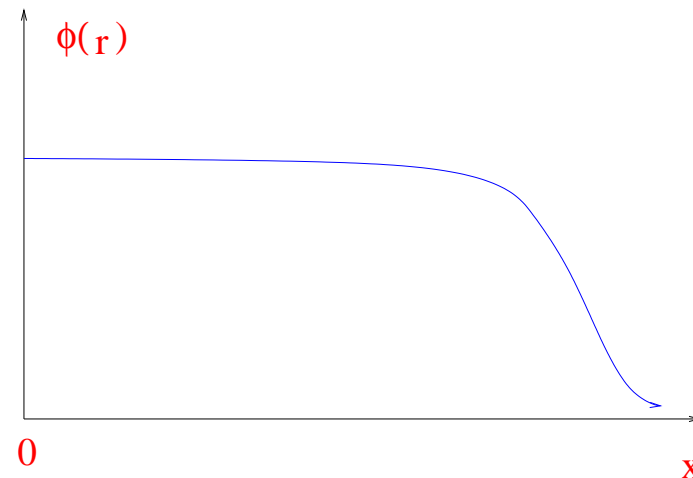
Solution: a spherically symmetric non-topological soliton

$$\phi(\mathbf{r}, t) = e^{i\omega t} \phi(\mathbf{r})$$

which exists provided the minimum of

$$\frac{U(\phi^* \phi)}{\phi^* \phi}$$

exists at some $\phi = \phi_0 \neq 0$.



Space profile of the scalar field of a Q-ball

What are Q-balls?

Particle physics models and Q-balls

Renormalizable field theory with one scalar field:

$$U(\phi^* \phi) = m^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

$$\frac{U(\phi^* \phi)}{\phi^* \phi} = m^2 + \lambda \phi^* \phi$$

The only minimum is at $\phi = 0 \rightarrow$ No Q-balls.

Experiment: The only known global symmetries are those associated with baryon number B and lepton number L . But, no scalar fields carrying B or L are known!

Supersymmetric extension of the standard model: MSSM

SUSY: each particle has a “superpartner”

spin	particle	superpartner	spin
$\frac{1}{2}$	quark	squark	0
$\frac{1}{2}$	lepton	slepton	0
0	higgs	higgsino	$\frac{1}{2}$
0	Z	zino	$\frac{1}{2}$

49 (!) scalar fields carrying baryon and lepton numbers →
many types of Q-ball solutions!

Crucial question : Are they stable?

$$M_Q = (M_s - \Delta M_s)Q$$

Q-balls are stable in the scalar sector (cannot decay into scalar particles) **but** may (?) decay quickly when fermions are taken into account:

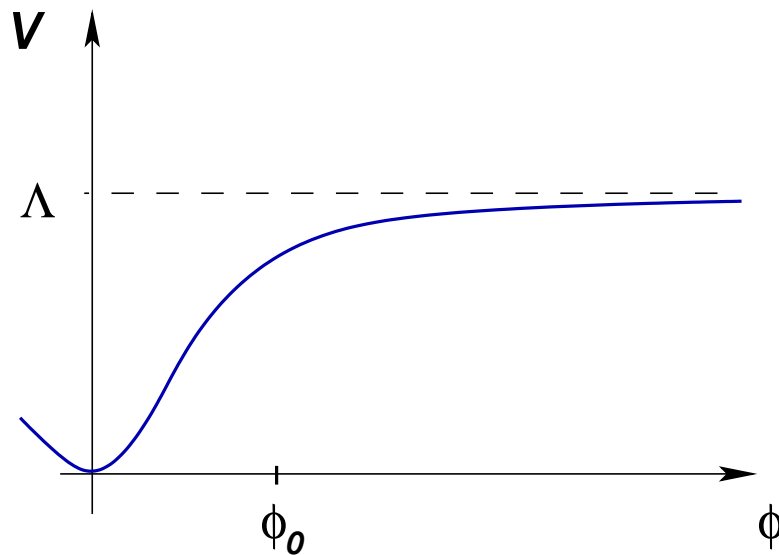
$$M_Q = (M_s - \Delta M_s)Q \gg m_p Q.$$

Possible process:

$$\tilde{q} + \tilde{q} \rightarrow q + q$$

SUSY theories with gauge-mediated supersymmetry breaking:

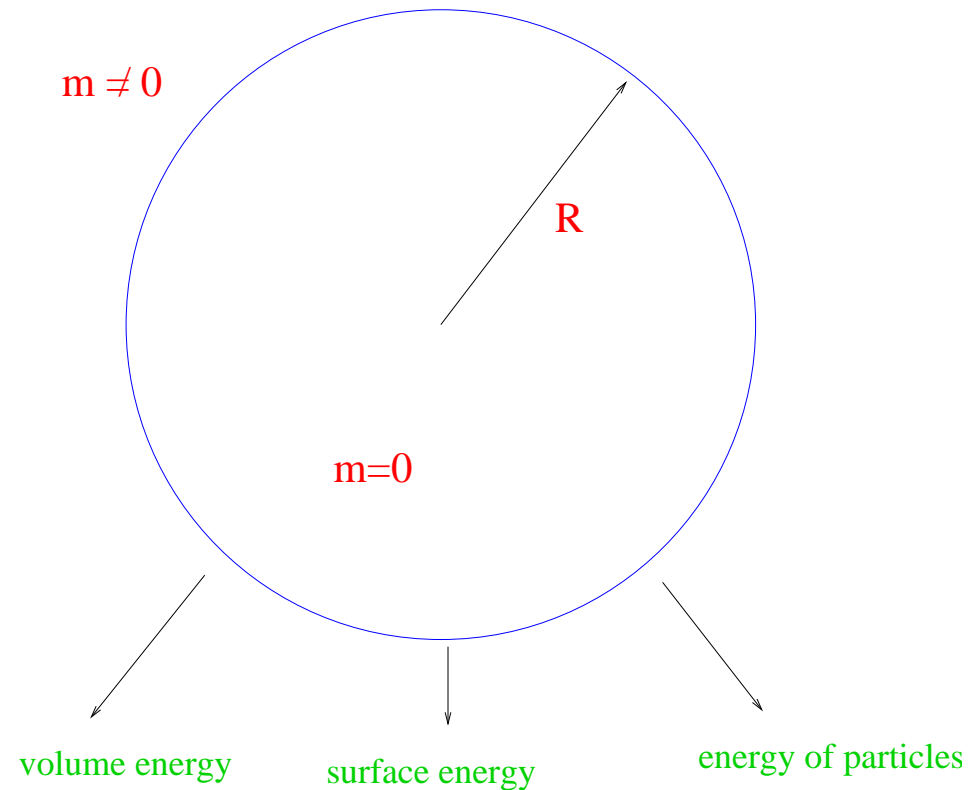
effective potential is flat along quite a number of directions in the space of scalar fields



This type of potential leads to existence of **stable** non-topological solitons, carrying baryon number

Typically, $\Lambda \sim 1 - 10 \text{ TeV}$, $\phi_0 \sim 1 - 10 \text{ TeV}$.

Mass of Q-ball:



$$M(R) = \frac{4}{3}\pi R^3 \Lambda^4 + 4\pi R^2 S + \pi \frac{Q^1}{R}$$

Minimization with respect to R gives:

$$\begin{aligned}\omega &\sim (\pi\sqrt{2})\Lambda Q^{-1/4}, \\ M_Q &\sim (4\pi\sqrt{2}/3)\Lambda Q^{3/4} \\ R_Q &\sim (1/\sqrt{2})\Lambda^{-1}Q^{1/4}\end{aligned}$$

If Q is the baryonic charge they are stable for ($\Lambda \sim 1\text{TeV}$)

$$Q > \left(\frac{4\pi\sqrt{2}\Lambda}{3m_p}\right)^4 \sim 10^{15}$$

found from $M_Q < m_p Q$.

Solutions to classical equations of motion for flat potential:

$$\phi(r, t) \simeq \phi_0 e^{i\omega t} \frac{\sin(\omega r)}{\omega r}, \text{ for } r < R_Q, \phi(r, t) \simeq 0, \text{ for } r > R_Q$$

SUSY theory is embedded finally into some GUT →

Baryon number is not conserved →

Q-balls are not stable

Questions:

- Q-ball lifetime

- Modes of decay

Method: add to the action higher-dimensional operators consistent with SUSY and look at the modified Q-ball properties.

The result depends on the structure of flat direction along which Q-ball is constructed

Degeneracy breaking

For large ϕ the degeneracy of potential is broken by higher order operators

$$V^n(\phi)_{\text{lifting}} \approx \lambda_n M^4 \left(\frac{\phi}{M} \right)^{n-1+m} \left(\frac{\phi^*}{M} \right)^{n-1-m}$$

$M \sim 10^{16}$ GeV is the scale of grand unification.

For $Q > Q_c$, (determined from $V^n(\phi)_{\text{lifting}} \sim \Lambda^4$)

$$Q_c \simeq \lambda_n^{-\frac{2}{n-1}} \left(\frac{M}{\Lambda} \right)^{\frac{4n-12}{n-1}}$$

properties of Q-balls change. m and n depend on the structure of flat direction of scalar potential.

Degeneracy breaking

- $m \neq 0$: explicit breaking of the global U(1) symmetry responsible for Q-ball existence \rightarrow strong baryon number non-conservation \rightarrow fast Q-ball decay.
- $m = 0$: Q-ball charge can grow beyond Q_c , but the dependence of the parameters of the Q-ball on Q is modified:

$$\phi_0 \sim \varphi_{\max}, \quad M \sim \mu Q, \quad \omega \sim \mu, \quad R \sim (\mu Q / \Lambda^4)^{1/3},$$

$$\text{where } \mu^2 \sim V(\varphi_{\max}) / \varphi_{\max}^2$$

Changing n from $n = 4$ to $n = 7$ the frequency ω ranges from **10 MeV** to **10^{-3} MeV** and the vev ϕ_0 from **10^7** to **10^{11} GeV**.

Baryon number violation - results

- Q-balls formed along $QUDE$, $QLUD$, $QLUDE$ directions cannot have baryon charge greater than $\sim 10^{17}$ and are short lived
- Q-balls formed along directions UE , QL , QU , LUE , UDE , QLE , QLD , QLU , $QU D$, $LUDE$, $QLDE$, $QLUE$, starting from $Q \sim 10^{17}$ have the following mass charge relation:

$$M \sim \omega_0 Q$$

with $\omega_0 \sim 10 MeV$ ($\Lambda \sim 1 TeV$). They decay by emitting neutrinos, antineutrinos and e^\pm pairs with typical energies $E \sim \omega_0$. Lifetime in general \gg universe age.

Baryon number violation - results

- other Q-balls (directions LD , LDE and some of UD , QUE , LUD) have the similar mass charge relation but with smaller $\omega_0 \sim 10^{-4} - 0.1$ MeV. They decay by emitting neutrinos and antineutrinos and their lifetime is huge compared with the universe age.

Very large Q-balls can become black holes,

$$Q_{\text{bh}} \simeq 2 \cdot 10^{53} \left(\frac{\text{TeV}}{\Lambda} \right)^2 \left(\frac{\text{MeV}}{\omega} \right) .$$

SUSY Q-balls as dark matter candidates

A most popular candidate to dark matter particle: neutralino

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It is difficult to detect neutralinos as they interact weakly with ordinary matter (WIMP)

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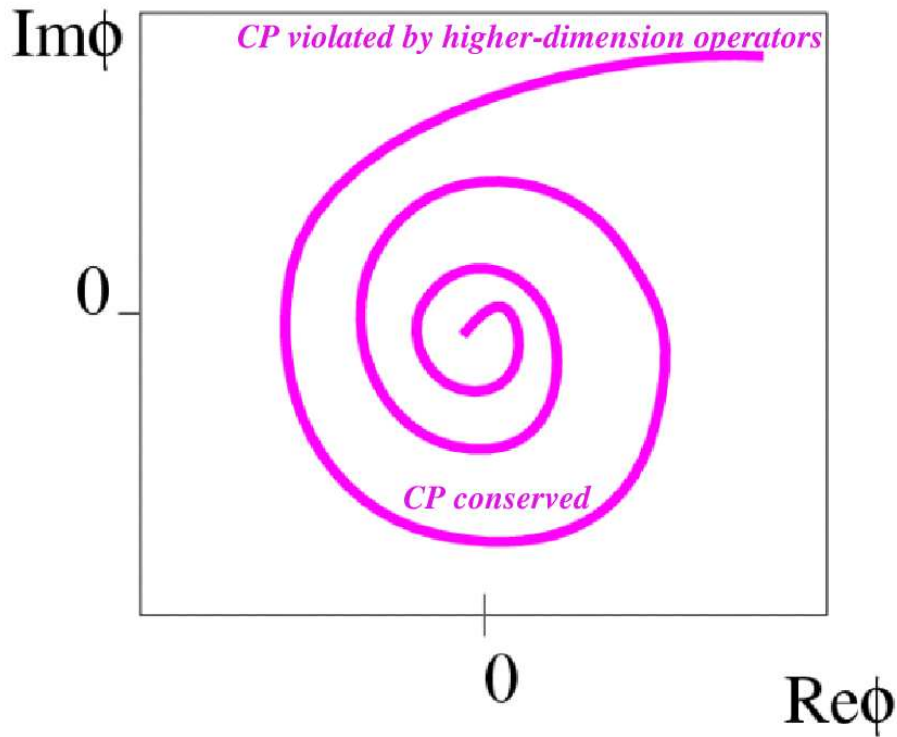
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It is difficult to detect Q-balls as their concentration is very small (super-heavy dark matter)



Affleck-Dine baryogenesis:

Inflation : Initial energy density $\sim M_{Pl}^4 \gg \Lambda^4 \rightarrow$ initial value of the scalar field is likely to be large, $\phi \sim M_{Pl} \rightarrow$ at large ϕ baryon number is not conserved and CP is broken \rightarrow Due to the classical evolution the universe acquires baryon number in the form of Affleck-Dine-condensate, $\phi \propto e^{i\omega t} \phi_0$

Cosmological production of Q-balls

Uniform configuration with $\phi \propto e^{i\omega t} \phi_0$ – Q-matter.

Observation: Q-balls are better than uniform Q-matter!

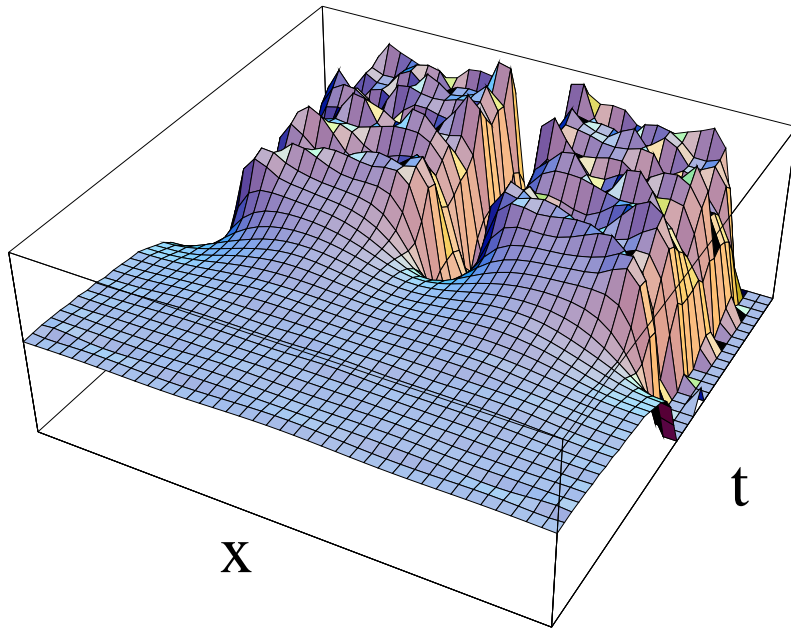
Charge density in Q-matter: n_Q , total charge: $n_Q V$

Mass of Q-matter: $\Lambda^4 V \gg \Lambda (n_Q V)^{3/4}$



Q-matter is unstable against Q-ball formation.

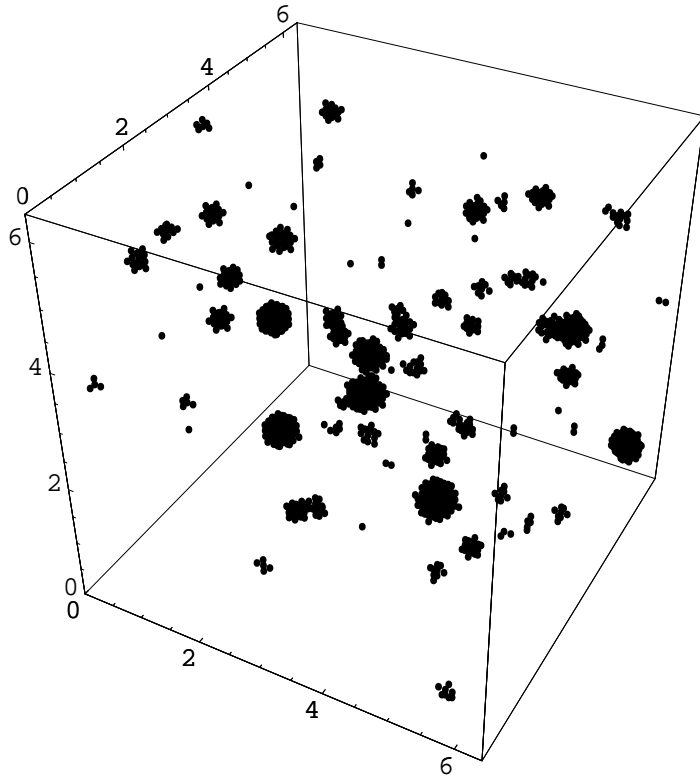
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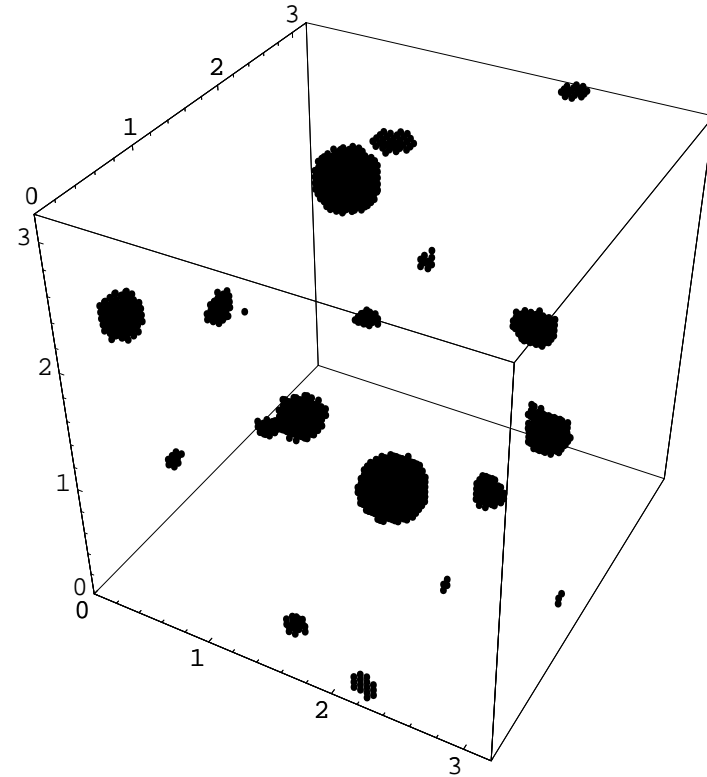
Instabilities in AD condensate



Q-ball formation



More than 40 Q-balls are formed, and the largest one has the charge with $Q \simeq 5.16 \times 10^{16}$



More than 10 Q-balls are formed, and the largest one has the charge with $Q \simeq 1.74 \times 10^{16}$

Cosmological production of Q-balls

How large a Q-ball could be? Charge density at temperature T :

$n_Q \sim \delta T^3$, $\delta \sim 10^{-10}$ is the baryon asymmetry.

Suppose that instability develops at $T = T_i$. Its size must be smaller than horizon at this time \implies

$$Q < \delta T^3 \left(\frac{T_i}{T} \right)^3 \left(\frac{M_{Pl}}{T_i^2} \right)^3 \sim \delta \left(\frac{M_{Pl}}{T_i} \right)^3$$

For $T \sim T_{EW} \sim 100 \text{ GeV}$ $Q < 10^{38}$.

Instability analysis: depends on the potential.

Result: Q-balls as large as $Q \sim 10^{26}$ are possible!

Dark matter and baryon asymmetry - the same source?!

AD condensate \rightarrow Q-ball formation \rightarrow Q-ball evaporation \rightarrow
baryon asymmetry \rightarrow an explanation why $\frac{\Omega_b}{\Omega_{DM}} \sim \frac{1}{7}$?

Computation:

$$\frac{\Omega_b}{\Omega_{DM}} \sim \frac{m_p M_{Pl}}{m_s^2 \sqrt{Q}}$$

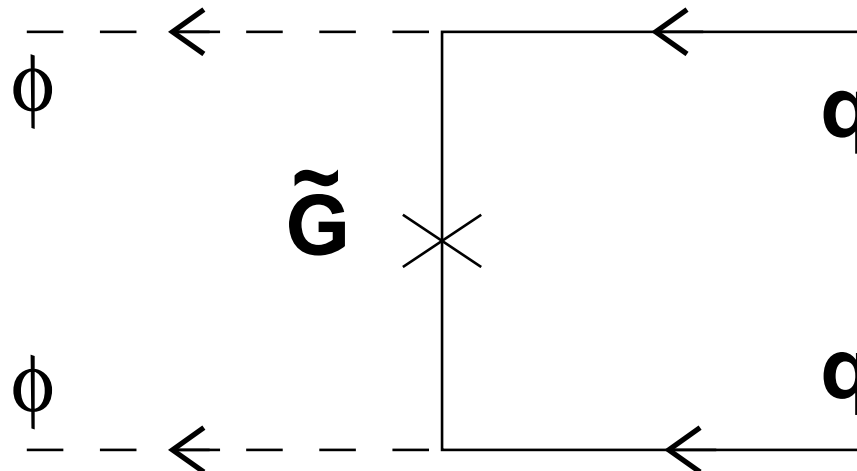
Dark matter can be baryonic ?!

Scalar field inside Q-ball generates Majorana mass for quarks:

$$m_q^M \sim g_s^2 \phi^2 \frac{M_M}{M_G^2}$$

(M_G -gluino mass) coming from

$$L_{eff} = g_s \bar{q} \lambda^a \phi \tilde{G}^a + M_M \tilde{G}^a \tilde{G}^a$$



Matter-antimatter conversion

Majorana mass inside Q-ball $\gg 1$ GeV \rightarrow baryons cannot propagate inside and are reflected as antiparticles from the surface of a Q-ball

Q-ball converts matter into antimatter with efficiency $\sim 100\%$:

$$n + Q(B) = \bar{n} + Q(B + 2)$$

$$\bar{n} + Q(B) = n + Q(B - 2)$$

Analogy - Andreev scattering in superconductors.

Unlimited new source of energy!

In principle, Q-balls can be distinguished from monopoles catalyzing proton decay, strangenets, nuclearites, etc

Assume: all DM in Q-balls with baryon number Q

$$n_Q \sim 3 \cdot 10^{-4} Q^{-\frac{3}{4}} \left(\frac{1 \text{ TeV}}{\Lambda} \right) \text{ cm}^{-3}$$

Q-ball flux is

$$F \sim 3 \cdot 10^{11} Q^{-\frac{3}{4}} \left(\frac{1 \text{ TeV}}{\Lambda} \right) \text{ cm}^{-2} \text{ yr}^{-1}$$

Strong interaction with matter: geometrical cross-section, $\sigma = \pi R_Q^2$

Signature: energy release $\sim 100 \text{ GeV/cm}$

Experimental limits from Baikal, Macro:

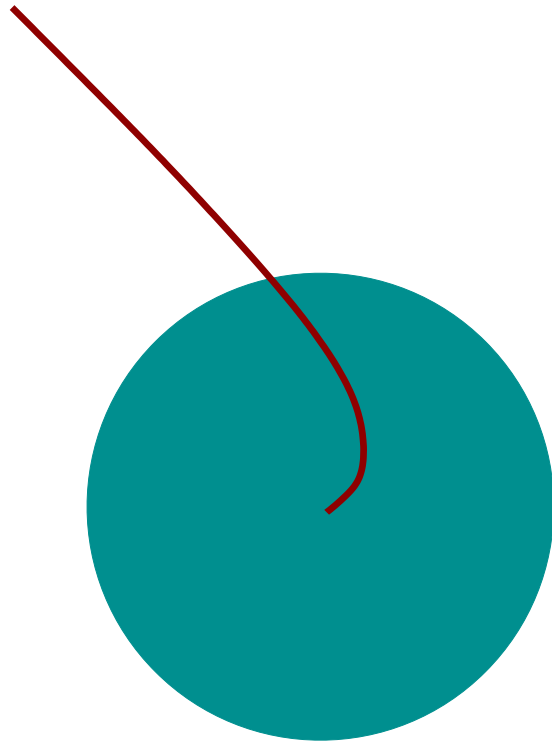
$$Q > 10^{23}$$

Astrophysical constraints

Q-balls do not accumulate in Earth and Sun: they are too heavy and go through without stopping:

$$\left(\frac{\delta v}{v}\right)_{Earth} \sim 10^{-7}, \quad \left(\frac{\delta v}{v}\right)_{Sun} \sim 10^{-5}$$

But neutron stars and white dwarfs stop Q-balls. For 10^8 years typical neutron star accumulates $\sim 10^{12}$ grams of Q-balls ($10^{-20} M_{NS}$).



Q-balls are trapped by
the neutron stars

Naive estimate of the neutron
star conversion rate to Q-ball:

$$\frac{dQ}{dt} \sim 4\pi R_Q^2 n_n$$

$$t \sim \frac{(\omega_c \phi_{max}^2)^{2/3}}{n_0} (Q_{bh})^{1/3} \sim 1\text{day??!}$$

Q-balls are excluded? (Neutron
stars are known to live at least 1
Gyr)

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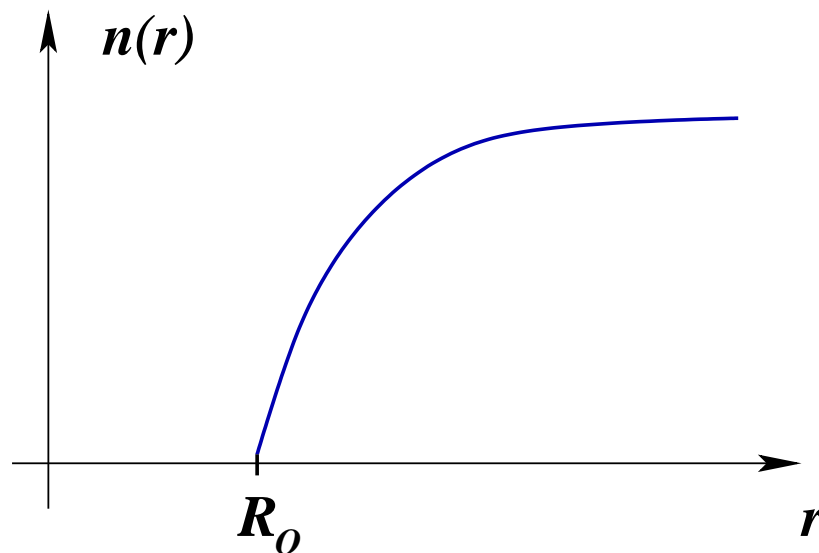
COSMIC KILLERS

Do squarks murder stars?



Diffusion and radiation pressure effects change the estimate completely

Diffusion:



$$\frac{dQ}{dt} \sim 4\pi R_Q D_n n_n$$

D_n - neutron diffusion length

$$t \sim \frac{\Lambda}{n_n D_n} (N_n)^{\frac{1}{4}} \sim 10^{13} \text{ years}$$

Eddington Accretion Limit: Radiation pressure force per nucleon:

$$F_{Edd} = \frac{\dot{M}\sigma}{4\pi r^2}.$$

is compensated by the gravity force $F_{Grav} = \frac{GM_Q m_p}{r^2}$.

Equilibrium at:

$$t = \frac{\sigma}{4\pi G\omega} \ln \frac{Q_{bh}}{Q_i} \approx 4 \times 10^{12} \text{years}.$$

Too long to be observed...

- Superheavy SUSY Q-balls, associated with most of flat directions are practically stable
- They can be used to convert easily matter into antimatter
- They can be copiously produced in the early universe
- Baryon asymmetry and dark matter can be related
- Q-balls can play a role of superheavy dark matter
- They have a very specific experimental signature