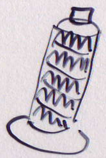
COLLECTIVE MODES AND PAIRING FLUCTUATIONS IN TRAPPED FERMI GASES


SCUOLA NORMALE SUPERIORE
PISA, ITALY


LPTMS ORSAY, FRANCE

ANNA MINGUZZI

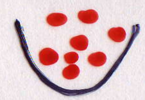
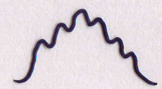
GEORG BRUUN (COPENHAGEN)

HUI HU, XIA-JI LIU (CHINA)

LUCIANO VIVERIT (TRENTO)

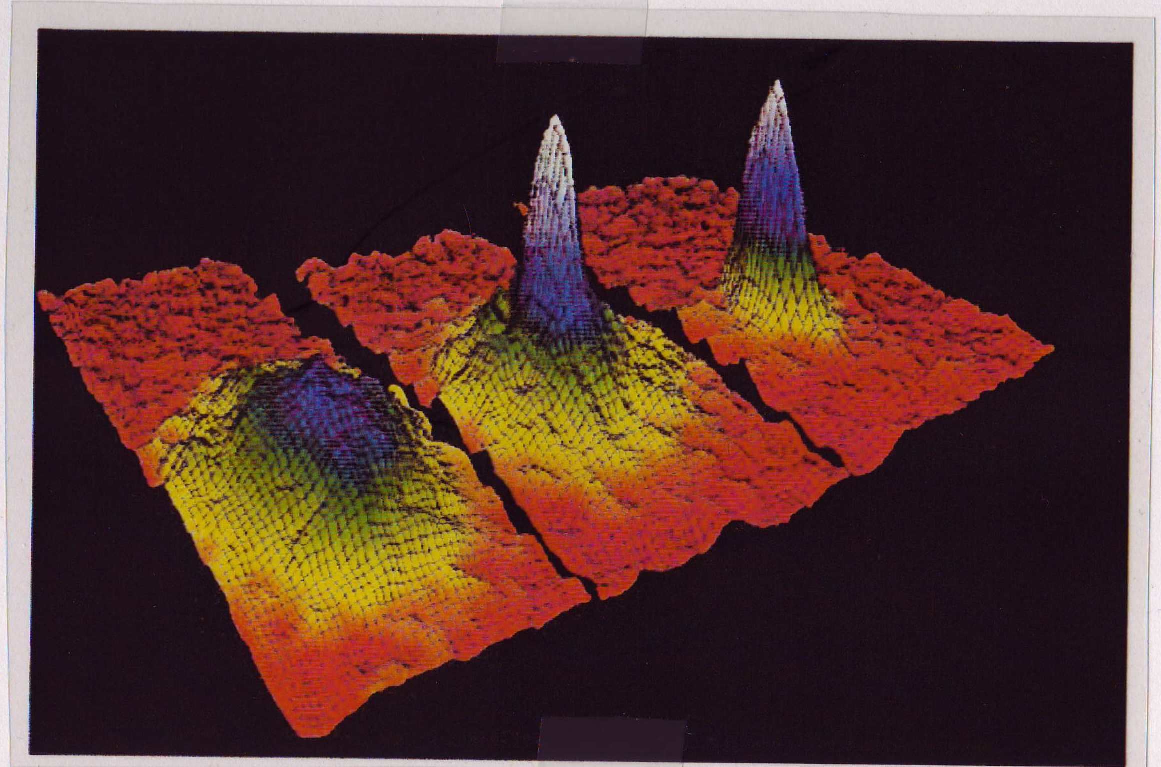
SARO FAZIO, MARIO TOSI (PISA)

PLAN

- INTRODUCTION ON COLD ATOMIC GASES 
- COLLECTIVE MODES IN THE BEC - BCS CROSSOVER 
- PAIRING AMPLITUDE FLUCTUATIONS IN THE BCS LIMIT 

QUANTUM FLUIDS WITH COLD ATOMS

* FROM LASER AND EVAPORATIVE COOLING



* OBSERVATION OF BOSE-EINSTEIN CONDENSATION IN METASTABLE GASES OF ALKALI ATOMS (1995)

$T \simeq 200\,nK \qquad n \simeq 10^{13}\,cm^{-3}$

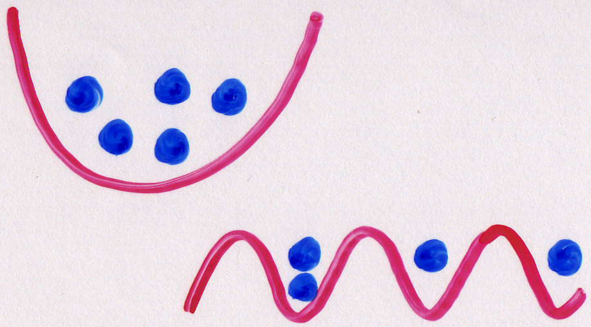
$N \simeq 10^{4} \div 10^{6}$

Anderson et al. Science 269 198 (1995)

* REALIZATION OF A DEGENERATE FERMI SEA $T \simeq 0.5\,T_F$ (1999)

* OBSERVATION OF A BOSE-EINSTEIN CONDENSATE OF FERMIONIC MOLECULES $T \simeq 0.1\,T_F$ (2003)

MODELLIZATION



MAGNETIC OR OPTICAL TRAP $\Rightarrow$

EXTERNAL CONFINEMENT
(HARMONIC WELL, LATTICE)

LOW TEMPERATURE, DILUTE GAS $\Rightarrow$

$T < 1\,\mu K$ $\quad n r_e^3 \ll 1$ $\quad r_e$, EFFECTIVE RANGE

INTERACTIONS CHARACTERIZED BY THE
S-WAVE SCATTERING LENGTH a

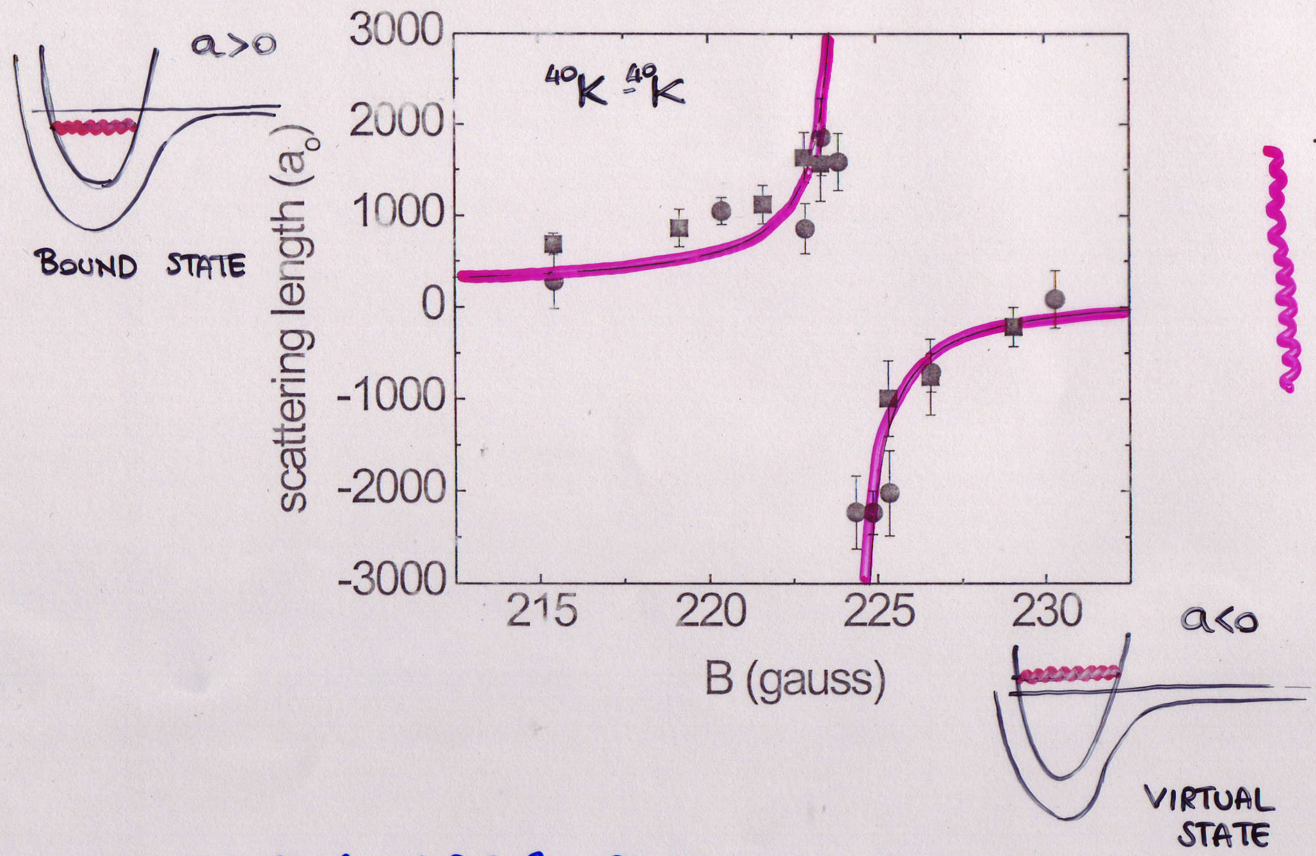
$$v(\vec{r}) = \frac{4\pi\hbar^2}{m} a\, \delta^3(r)$$ CONTACT PSEUDOPOTENTIAL

TYPICALLY $a \gg r_e$

IN EARLY EXPERIMENTS $n a^3 \ll 1$

FESHBACH RESONANCES

* THE SCATTERING LENGTH CAN BE TUNED BY APPLYING A MAGNETIC FIELD



THE RESONANCE IS DUE TO THE APPROACH OF A TWO-BODY BOUND STATE TO THE CONTINUUM

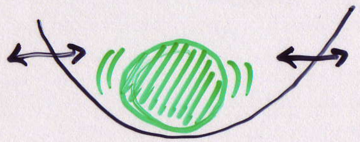
! THE FERMI GAS IS STABLE ALSO FOR $k_F|a| \gtrsim 1$

⇝ STRONGLY CORRELATED REGIME

C.A. Regal and D.S. Jin PRL 90 230404 (2003)

EXPERIMENTAL PROBE OF COLLECTIVE MODES

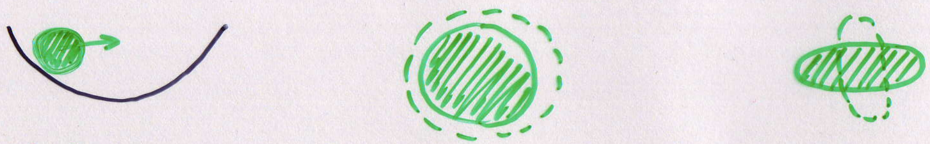
→ LOW-ENERGY MODES



- APPLY A TIME-DEPENDENT PERTURBATION TO THE TRAP
- FOLLOW THE TIME EVOLUTION $\langle \vec{r}(t) \rangle$ OR $\langle r^2(t) \rangle$

⇝ ONE OF THE MOST ACCURATE MEASUREMENTS

SEVERAL SYMMETRIES ARE POSSIBLE



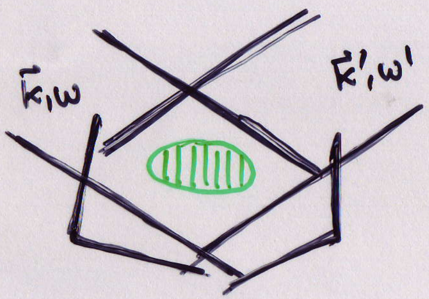
DIPOLE

BREATHING

QUADRUPOLE

→ BULK MODES

$q > 1/R$ R, SIZE OF THE CLOUD



"BRAGG SPECTROSCOPY"

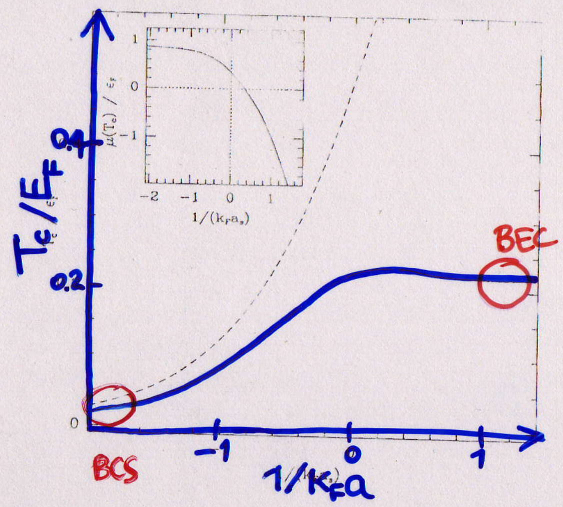
TRANSFER ENERGY $\Omega = \omega - \omega'$ AND MOMENTUM $\vec{q} = \vec{k} - \vec{k}'$ TO ATOMS

→ MEASURE THE DYNAMICAL STRUCTURE FACTOR $S(\vec{q},\Omega)$

SUPERFLUID FERMI GAS

* TWO-COMPONENT FERMI GAS $\uparrow,\downarrow$, INTERCOMPONENT S-WAVE INTERACTIONS PARAMETRIZED BY $a_{\uparrow\downarrow}$

AT LOW TEMPERATURES THE GAS IS EXPECTED TO UNDERGO A SUPERFLUID TRANSITION



$$\Delta(r) = g_{\uparrow\downarrow} \langle \hat{\psi}_{\uparrow}(r) \hat{\psi}_{\downarrow}(r) \rangle$$ ORDER PARAMETER

"CROSSOVER BCS-BEC" Leggett (1980), Nozières Schmitt-Rink (1985)

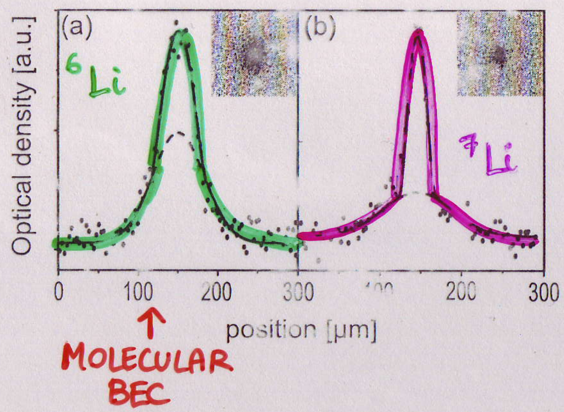
REGIME BEC CONDENSATE OF MOLECULES

EXPERIMENTALLY OBSERVED

REGIME BCS LARGE INTERTWINED PAIRS

$\Delta \ll E_F$ THE TRANSITION DOES NOT AFFECT THE DENSITY PROFILES

→ HOW TO DETECT IT?



T. Bourdel et al., PRL 93 050401 (2004)

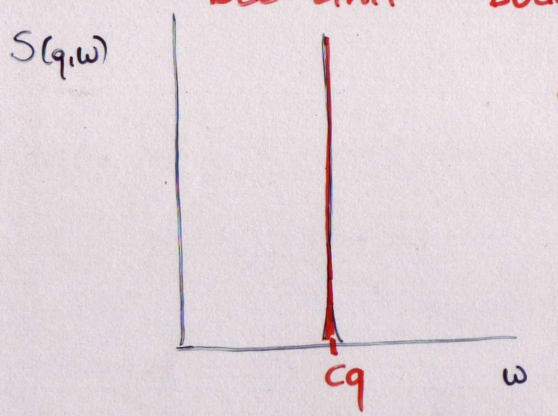
COLLECTIVE MODES FOR A HOMOGENEOUS SUPERFLUID

* SPECTRUM OF COLLECTIVE MODES : THE DYNAMIC STRUCTURE FACTOR $S(\vec{q},\omega)$ IS OBTAINED FROM THE DENSITY-DENSITY RESPONSE FUNCTION

→ CALCULATION IN THE BCS LIMIT THROUGH A GENERALIZED RANDOM-PHASE APPROXIMATION

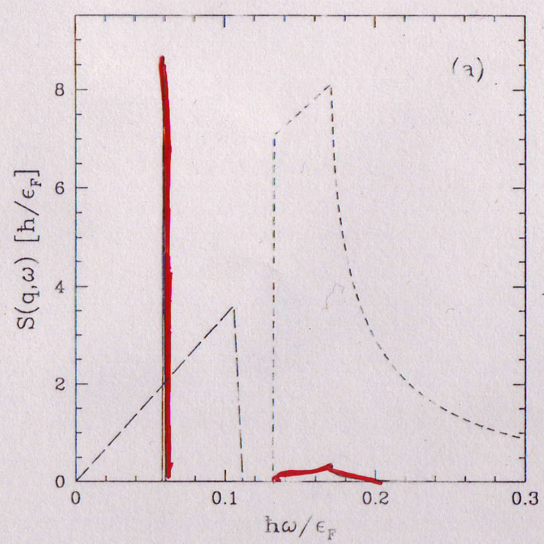
! FOR A SUPERFLUID DESCRIBED BY A COMPLEX ORDER PARAMETER A SOFT MODE IS EXPECTED FOR $q \to 0$ (GOLDSTONE THEOREM)

BEC LIMIT BOGOLIUBOV SOUND



$c = \sqrt{gn/m}$

BCS LIMIT ANDERSON-BOGOLIUBOV SOUND



$c = \frac{v_F}{\sqrt{3}}$

A.M., G. Ferrari, Y. Castin, EPJD 17 49 (2001)

COLLECTIVE MODES OF A TRAPPED SUPERFLUID

* WE USE THE EQUATIONS OF MOTION OF GENERALIZED HYDRODYNAMICS

$$\partial_t n + \nabla \cdot (n\vec{v}) = 0$$

$$m\,\partial_t \vec{v} + \nabla\left(\mu(n) + V_{ext} + \frac{1}{2} m v^2\right) = 0$$

$V_{ext} \rightarrow$ HARMONIC CONFINEMENT

$$\mu(n) = \begin{cases} g n & \text{BEC LIMIT} \\ \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} & \text{BCS LIMIT} \end{cases}$$

⇒ QUANTIZATION OF THE BOGOLIUBOV AND ANDERSON-BOGOLIUBOV SOUNDS

(SPHERICAL TRAP)

BEC LIMIT

$$\omega_{n\ell} = \omega_0 \sqrt{2n^2 + 2n\ell + 3n + \ell}$$

S. Stringari, PRL 77, 2360 (1996)

BCS LIMIT

$$\omega_{n\ell} = \omega_0 \sqrt{\ell + 4n(n+\ell+2)/3}$$

HOLDS ALSO AT THE "UNITARITY" LIMIT $1/k_F a = 0$

WHERE $E(n) = \eta\, E_{\substack{FREE\\FERMI}}(n)$ $\quad \eta \simeq 0.444$

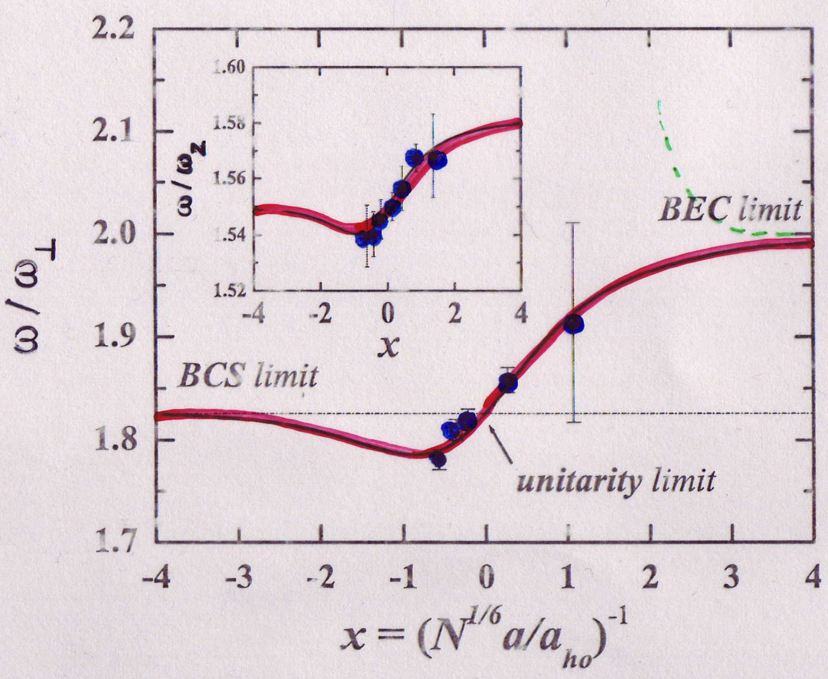
M.A. Baranov and D.S. Petrov PRA 62 041601 (2000)

M. Amoruso, I. Meccoli, A.M., MP Tosi, EPJD 7 441 (1999)

BREATHING MODES IN THE BEC-BCS CROSSOVER

- EQUATION OF STATE $\mu(n)$ FROM THE MEAN FIELD GAP AND NUMBER EQUATIONS IN THE CROSSOVER
- SCALING ANSATZ

$$n(\vec{r},t) = n_0\left(x_i/b_i(t)\right) / \prod_j b_j(t)$$
$$V_i(\vec{r},t) = \dot{b}_i(t)\, x_i / b_i(t)$$



AXIAL AND TRANSVERSE BREATHING MODES

→ DEPARTURE FROM POINT-LIKE BOSON APPROXIMATION (- - -)

⇒ GOOD AGREEMENT WITH EXPERIMENT (●)

AT $T=0$ THE CORRECTIONS BEYOND MEAN FIELD ARE EXPECTED TO BE SMALL

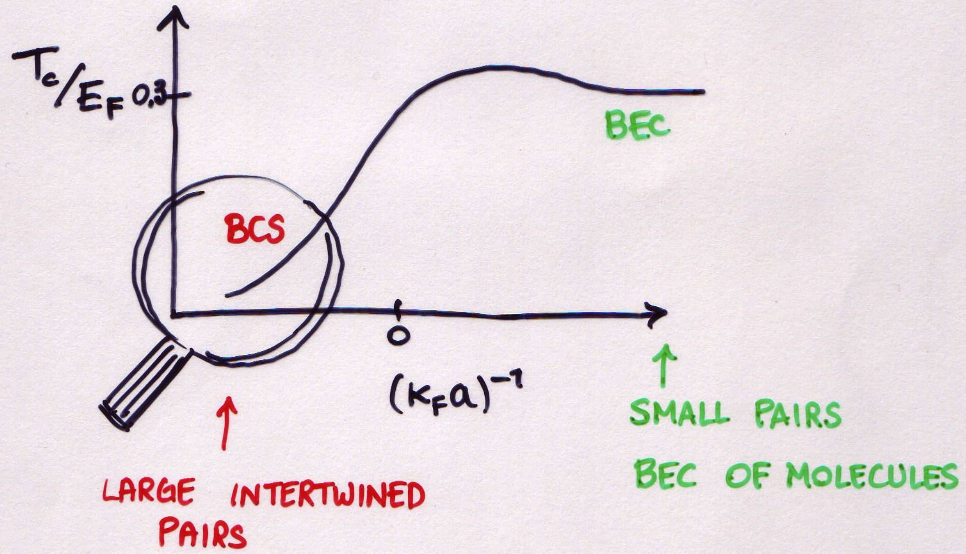
Hui Hu, A.M., Xia-Ji Liu, M.P. Tosi PRL 93 190403 (2004)

SUPERFLUID FERMI GASES

* A TWO-COMPONENT FERMI GAS MAY UNDERGO A SUPERFLUID TRANSITION

  ⇝ MAJOR EXPERIMENTAL EFFORT

* BEC - BCS CROSSOVER



Regal et al. Nature 404 47 (2003)
Jochim et al. Science 302 2101 (2003)
Regal et al. PRL 92 250401 (2004)

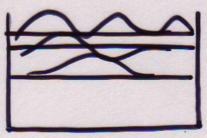
PAIRING FLUCTUATIONS:
OUR AIM

FOR A SMALL FERMI CLOUD UNDER HARMONIC CONFINEMENT

WE WANT TO UNDERSTAND

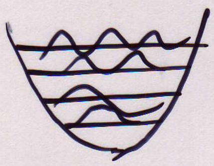
- THE MAGNITUDE OF PAIRING FLUCTUATIONS
- THE WIDTH OF THE FLUCTUATION REGION
- THE SPATIAL DEPENDENCE OF PAIRING FLUCTUATIONS
- HOW THEY CAN BE OBSERVED

BCS REGIME



UNIFORM SYSTEM:

PAIRING AMONG $|\vec{q},\uparrow\rangle$ AND $|-\vec{q},\downarrow\rangle$



(SMALL) HARMONICALLY TRAPPED SYSTEM:

PAIRING AMONG $|n,\ell,m;\uparrow\rangle$ AND $|n,\ell,-m;\downarrow\rangle$

MEAN-FIELD BCS THEORY

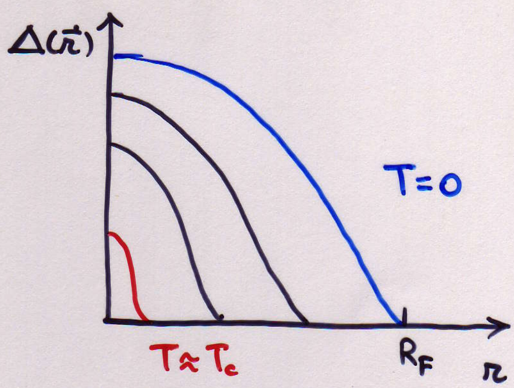
$$\Psi_\uparrow(\vec{r}) = \sum_\nu u_\nu(\vec{r})\, b_{\nu\uparrow} - v_\nu^*(r)\, b^+_{\nu\downarrow}$$

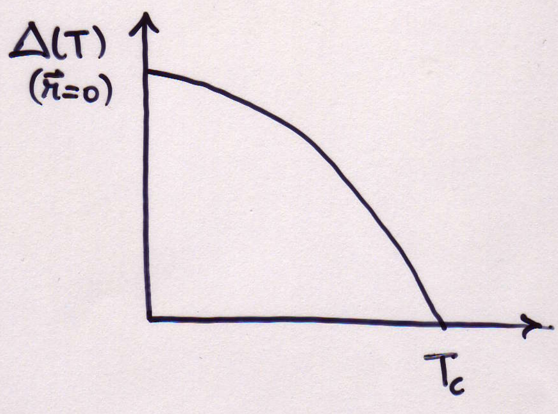
$u_\nu(\vec{r}), v_\nu(\vec{r})$ SATISFY THE BOGOLIUBOV - DE GENNES EQUATIONS WITH ENERGY E_ν

ORDER PARAMETER

$$\Delta(\vec{r}) = g\langle\Psi_\uparrow(\vec{r})\Psi_\downarrow(\vec{r})\rangle$$

$$= -g\sum_\nu u_\nu(\vec{r})v_\nu(\vec{r})\,(1-2f(E_\nu))$$





Stoof et al. PRL 76 10 (1996)

Baranov and Petrov PRA 58 801 (1998)

Bruun et at. EPJD 7 433 (1999)

CLOSE TO THE CRITICAL TEMPERATURE

$\Rightarrow$ MEAN-FIELD THEORY FAILS

WE NEED TO TAKE INTO ACCOUNT FLUCTUATIONS

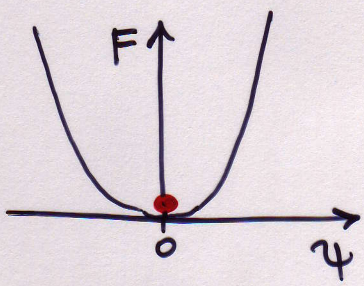
GINZBURG - LANDAU THEORY

* FREE-ENERGY DENSITY $\quad \psi(\vec{x}) \propto \Delta(\vec{x})$

$$F(\psi) = \alpha(T)\,|\psi|^2 + \beta\,|\psi|^4 + \frac{\hbar^2}{2m}\,|\nabla\psi|^2 + \cdots$$

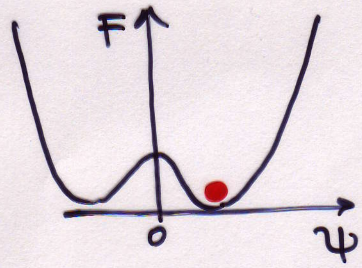
WITH $\quad \alpha(T) = \bar{\alpha}\,(T/T_c - 1)$

$T > T_c$



$\langle\psi\rangle = 0$

$T < T_c$



$\langle\psi\rangle \neq 0$

BUT $\langle\psi^2\rangle \neq 0$

$\Downarrow$

THERMAL FLUCTUATIONS

$$Z = e^{-FV/k_B T}$$

$$\langle F - F_0\rangle \simeq \frac{1}{2}\frac{\partial^2 F}{\partial\psi^2}\langle\delta\psi^2\rangle = k_B T$$

$$\Rightarrow \quad \langle\psi^2\rangle = \frac{k_B T}{V\bar{\alpha}}\,\frac{1}{(T/T_c - 1)}$$

FLUCTUATIONS

* PAIRING FLUCTUATIONS SIGNAL THE ONSET OF COOPER PAIRING ABOVE T_c

e.g. MOMENTUM DISTRIBUTION, DENSITY CORRELATIONS etc.

? HOW LARGE IS THIS EFFECT ?

$$\frac{\langle (\delta\Delta)^2 \rangle}{[\Delta(T=0)]^2} \simeq \mathcal{A} \frac{1}{T/T_c - 1}$$

ω = TRAP FREQUENCY

WITH AMPLITUDE $\mathcal{A} \approx \frac{\hbar\omega}{\Delta(0)} \frac{1}{N^{2/3}} \sim \frac{\xi}{R} \frac{1}{N^{2/3}}$

$\xi = \hbar^2 k_F / \Delta(0)$
COHERENCE LENGTH

R = SIZE OF CLOUD

$\Rightarrow$ FLUCTUATIONS ARE IMPORTANT

IN SMALL SYSTEMS : $\xi \gtrsim R$

$N \lesssim 500$ (SMALL MAGNETIC TRAPS, OPTICAL LATTICES)

REMARK

$\xi \gtrsim R$ IMPLIES $\Delta < \hbar\omega$: "INTRASHELL REGIME"

NO VORTICES

MICROSCOPIC MODEL

FOR THE FERMI GAS UNDER EXTERNAL CONFINEMENT

HAMILTONIAN

$$\mathcal{H} = \sum_\sigma \int d^3r \, \psi_\sigma^+ \left(-\frac{\nabla^2}{2m} + V_{ext}(\vec{r}) - \mu \right) \psi_\sigma$$

$$+ g \int d^3r \, \psi_\uparrow^+ \psi_\downarrow^+ \psi_\downarrow \psi_\uparrow$$

$g < 0$ WE USE THE CONTACT PSEUDOPOTENTIAL

$\rightsquigarrow$ REGULARIZATION

EFFECTIVE ACTION $S[\Delta, \Delta^*]$

FUNCTIONAL INTEGRAL APPROACH

$\Delta(\vec{x}, \tau)$ AUXILIARY FIELD COUPLED TO $\psi_\uparrow \psi_\downarrow$

($\rightarrow$ HUBBARD - STRATONOVICH TRANSFORMATION)

PARTITION FUNCTION

$$Z = Z_{\substack{\text{normal} \\ \text{gas}}} \int d^2[\Delta] \, e^{-S[\Delta, \Delta^*]/\hbar}$$

(WE HAVE NEGLECTED DENSITY FLUCTUATIONS)

PAIRING FLUCTUATIONS

DEFINITION

$$\langle |\Delta^2(r)| \rangle = Z^{-1} \int d^2[\Delta] \; |\Delta|^2 \; e^{-S[\Delta,\Delta^*]/\hbar}$$

FOR $T > T_c$, IN THE QUADRATIC APPROXIMATION FOR S

WE EXPAND IN NORMAL MODES $\{ \chi_\nu(\vec{r}), \alpha_\nu \}$

$$\Delta(\vec{r}) = \sum_\nu \chi_\nu(\vec{r}) \, \vec{\Delta}_\nu$$

$$\int d^3r_1 \; A(r_1, r_2) \, \chi_\nu(r_1) = \alpha_\nu \, \chi_\nu(r_2)$$ "NATURAL ORBITALS"

$$\Rightarrow \quad S^{(2)} = \hbar\beta \sum_\nu \alpha_\nu \, |\vec{\Delta}_\nu|^2$$

- CLOSE TO T_c THE SMALLEST EIGENVALUE α_0 HAS THE LARGEST EFFECT

$$\boxed{\langle |\Delta^2(r)| \rangle = \frac{k_B T_c}{\alpha_0(T)} \, |\chi_0(r)|^2}$$

[VALID FOR A SMALL SYSTEM]

SPATIAL DEPENDENCE OF THE PAIRING FLUCTUATIONS

* FERMI GAS UNDER ISOTROPIC HARMONIC CONFINEMENT

→ IN THE SMALL-SYSTEM LIMIT :
ANALYTIC SOLUTION OF THE EIGENVALUE EQUATION

$$\int d^3r_1 \; A(r_1, r_2)\, \chi_0(r_1) = \alpha_0 \, \chi_0(r_2)$$

- neglect Hartree fields
- assume pairing at the Fermi surface
- take angular averages

$$\chi_0(r) \propto \underbrace{\sum_{\ell} (2\ell+1)\, |R_{n_F \ell}(r)|^2}_{\text{RADIAL EIGENFUNCTIONS OF THE HARMONIC OSCILLATOR}} \approx \underbrace{\sqrt{1-(r/R_{TF})^2}}_{\text{THOMAS-FERMI APPROXIMATION}}$$

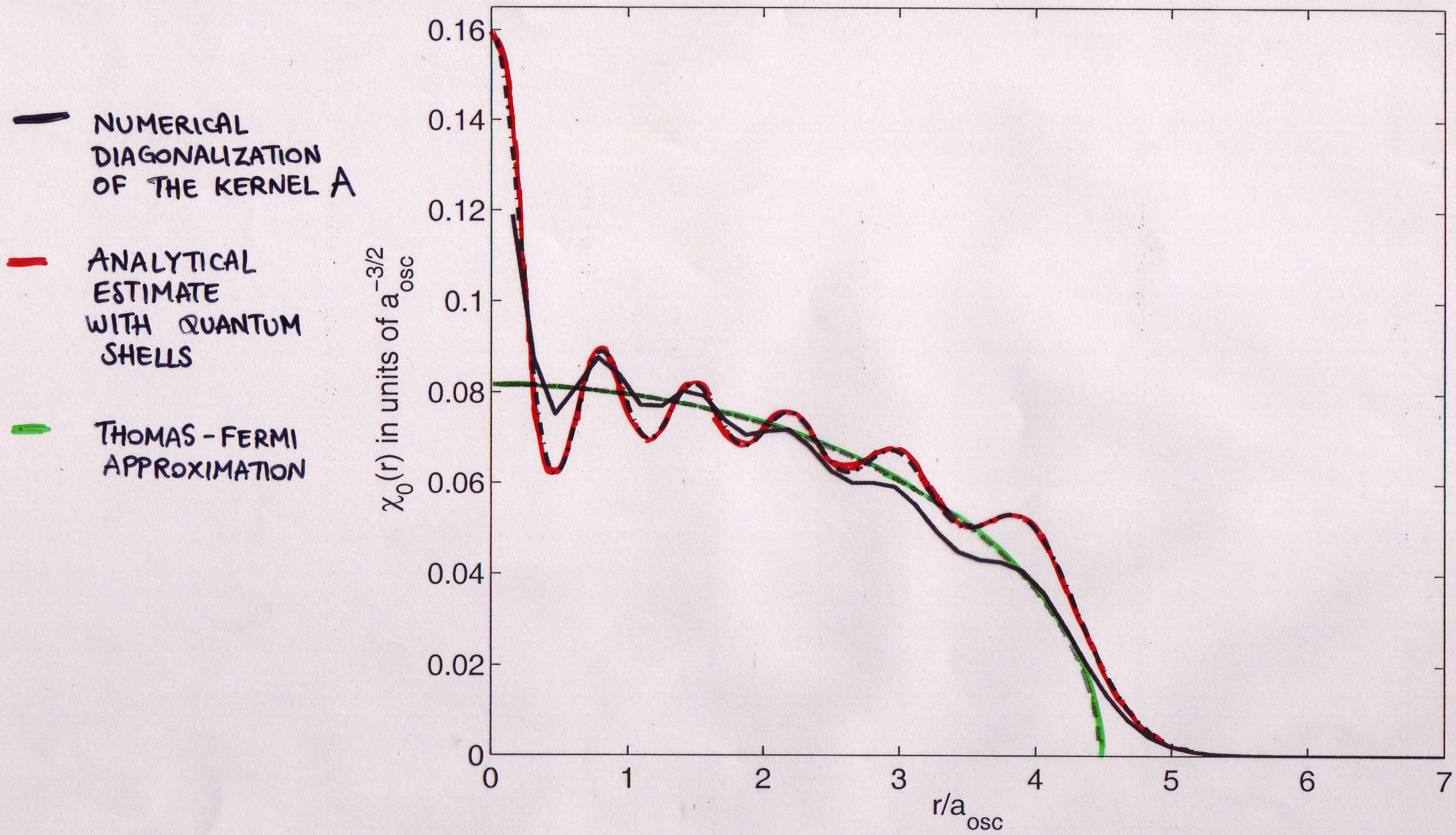
→ FLUCTUATIONS OCCUR MOSTLY AT THE TRAP CENTER

⇝ "ONSET OF CONDENSATION"

→ FLUCTUATIONS EXTEND UP TO THE BORDERS OF THE CLOUD

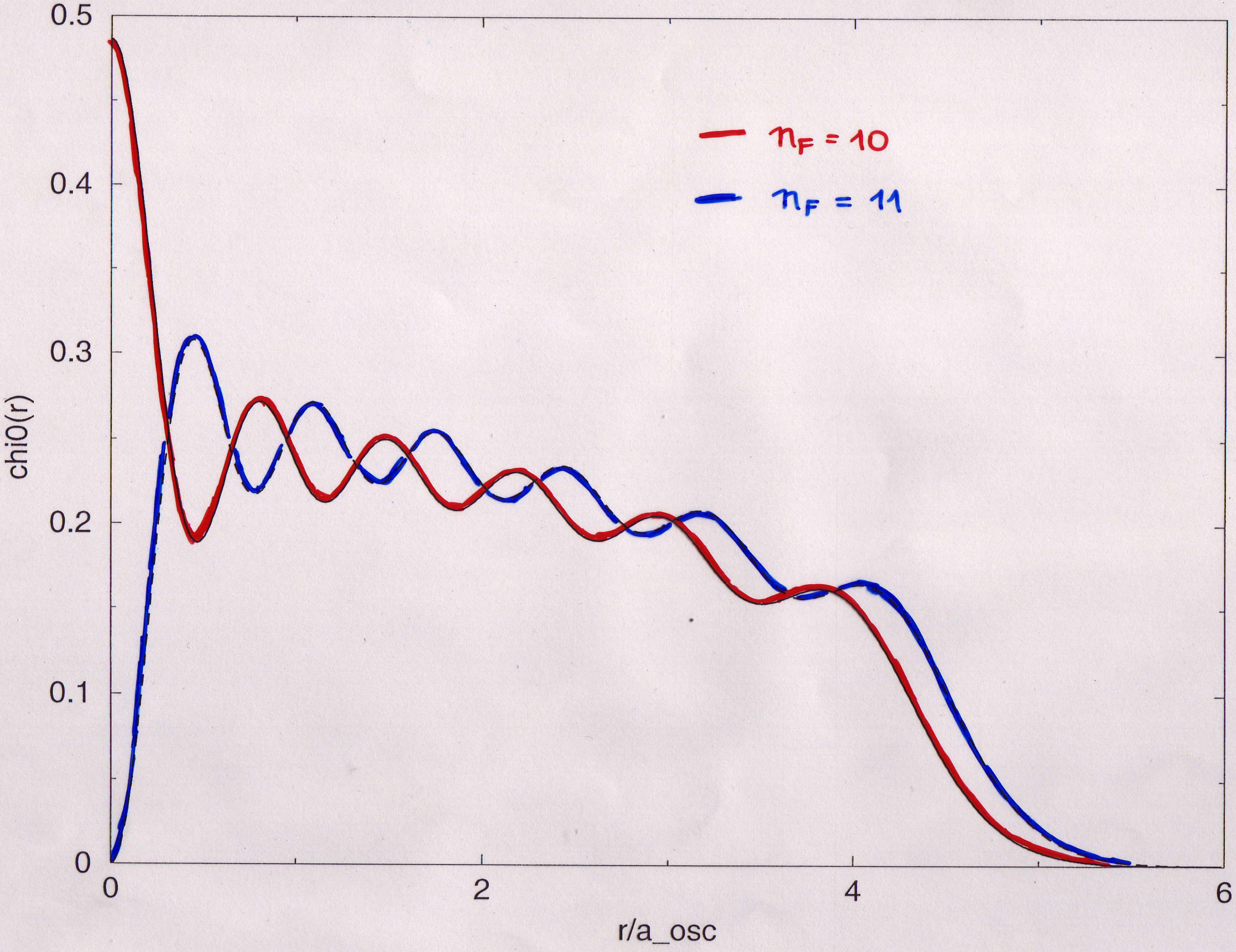
SPATIAL DEPENDENCE OF THE PAIRING FLUCTUATIONS

$N \simeq 500$ PARTICLES , $n_F = 10$, $|g| = 0.7\ \hbar\omega\ a_{osc}^3$



L. Viverit, G. Bruun, A.M. and R. Fazio PRL 93 110406 (2004)

PARITY EFFECT n_F EVEN / ODD



TEMPERATURE DEPENDENCE OF THE PAIRING FLUCTUATIONS

TO QUADRATIC ORDER $\langle \Delta^2(\tau=0) \rangle \propto \dfrac{1}{\alpha_0(T)}$

<u>BUT</u> $\alpha_0(T=T_c)=0$

"THOULESS CRITERION" FOR THE ONSET OF COOPER PAIRING

(DEFINITION OF THE MEAN-FIELD T_c)

$\alpha_0(T) \approx (T/T_c - 1) \Rightarrow$ SPURIOUS DIVERGENCE OF $\langle \Delta^2 \rangle$

* THE QUARTIC-ORDER CORRECTIONS IN THE EFFECTIVE ACTION CURE THE DIVERGENCE

FOR SMALL SYSTEMS

CALCULATION IN THE SINGLE-MODE APPROXIMATION

* THE MAGNITUDE OF THE CRITICAL REGION DEPENDS ON n_F ONLY

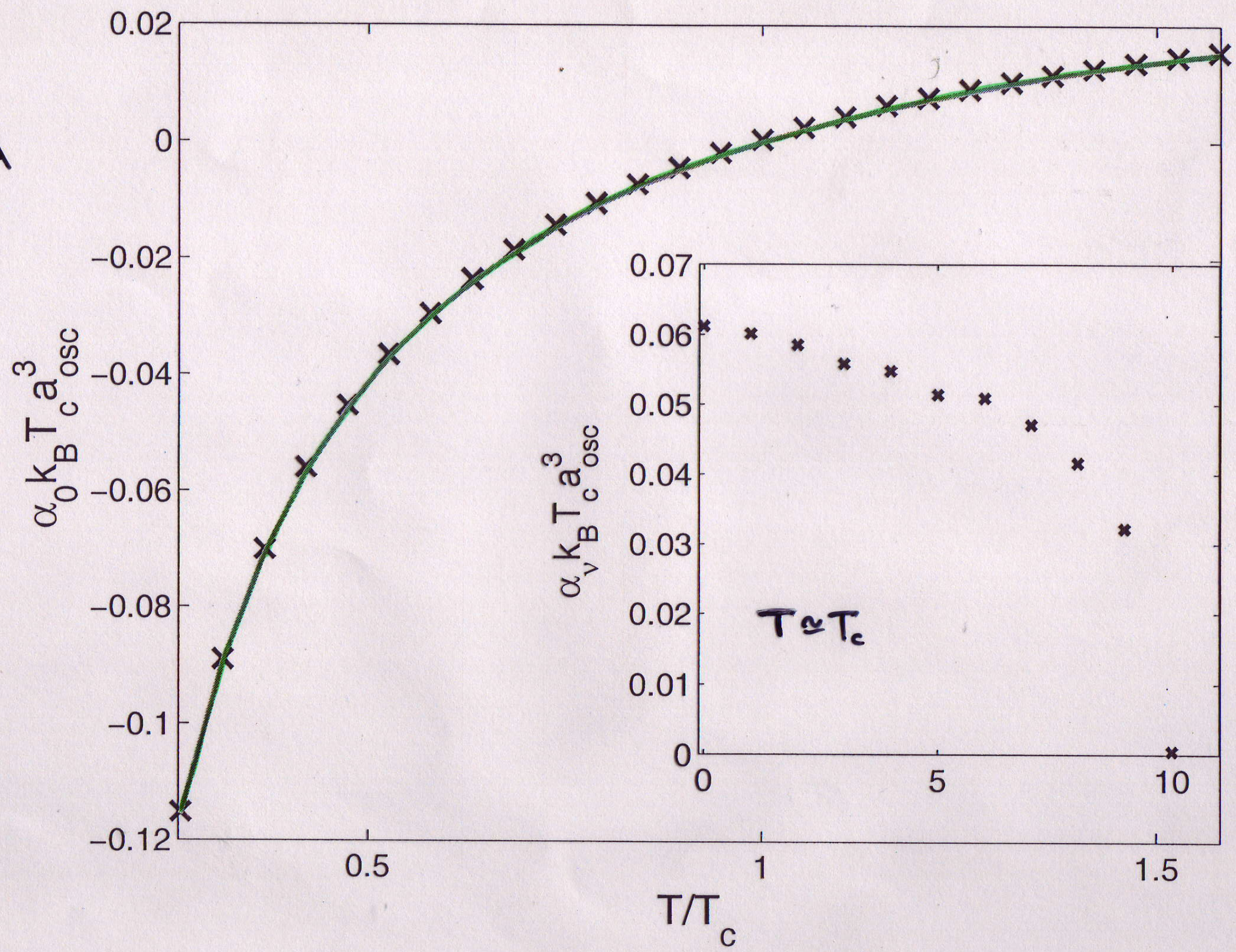
GINZBURG CRITERION: $\dfrac{\delta T}{T_c} \sim \dfrac{2}{(n_F+1)(n_F+2)}$

EIGENVALUES OF $A(\vec{r}_1, \vec{r}_2)$

$N \simeq 500$ PARTICLES , $n_F = 10$, $|g| = 0.7\, \hbar\omega\, a_{ho}^3$

× NUMERICAL DIAGONALIZATION OF REGULARIZED A

— ANALYTICAL ESTIMATE



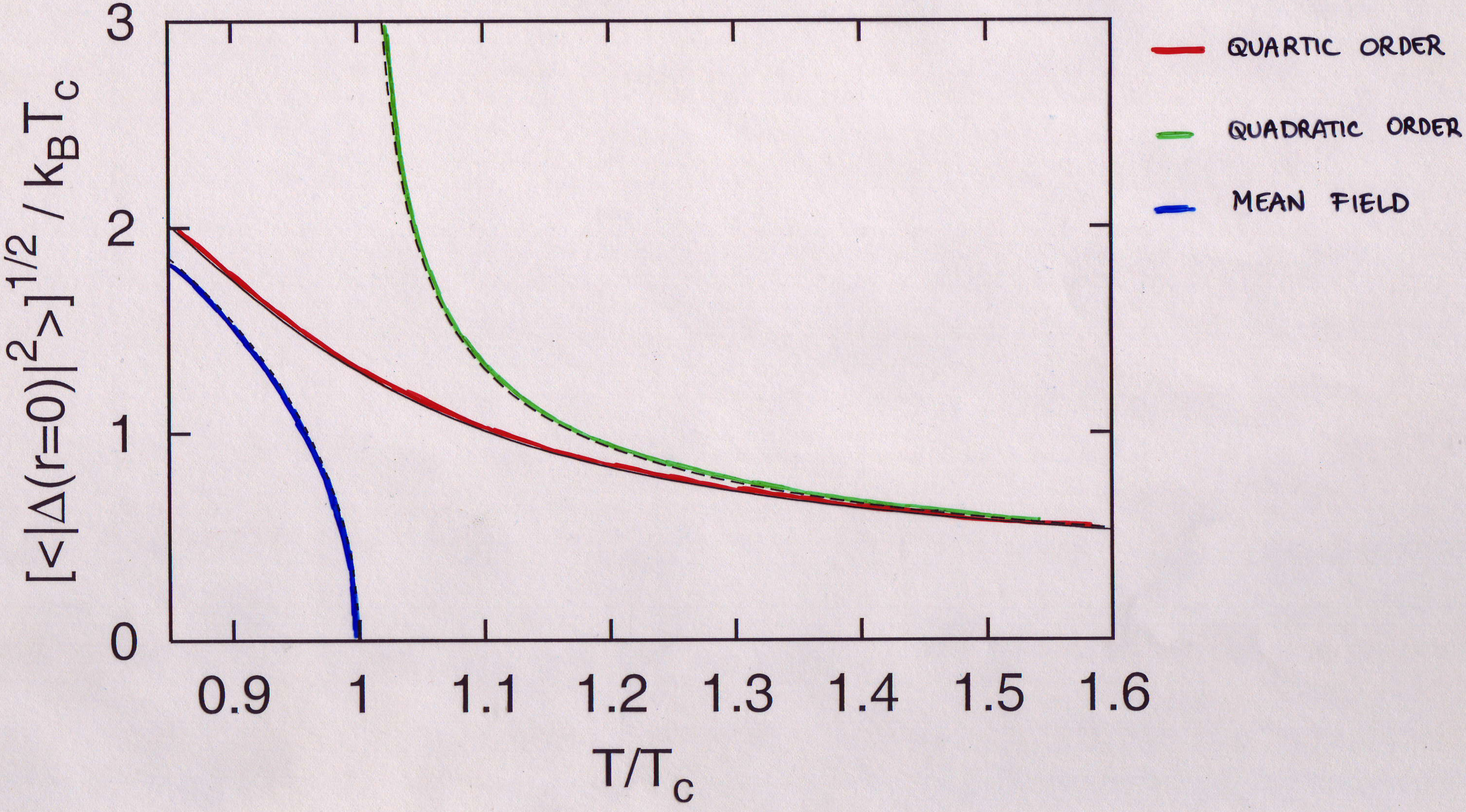
THE LOWEST EIGENVALUE IS WELL SEPARATED

$\Downarrow$

SINGLE-MODE APPROXIMATION

TEMPERATURE DEPENDENCE OF THE PAIRING FLUCTUATIONS

$N \simeq 500$ PARTICLES , $n_F = 10$, $|g| = 0.7\ \hbar\omega\ a_{osc}^3$



DENSITY CORRELATIONS

- FOR A DILUTE GAS THE SPIN-RESOLVED DENSITY CORRELATOR PROBES THE PAIRING AMPLITUDE AND ITS FLUCTUATIONS

e.g. AT MEAN-FIELD LEVEL

$$\langle n_\uparrow n_\downarrow \rangle - \langle n_\uparrow \rangle \langle n_\downarrow \rangle = \langle \psi_\uparrow^+ \psi_\downarrow^+ \rangle \langle \psi_\uparrow \psi_\downarrow \rangle = |\Delta|^2$$

* FOR A HOMOGENEOUS SYSTEM "ANOMALOUS" CORRELATIONS OCCUR AT $\vec{q}, -\vec{q}$:

$$\langle \delta n_{\vec{q}\uparrow} \, \delta n_{-\vec{q}\downarrow} \rangle \propto \langle \Delta^2 \rangle$$

! MEAN-FIELD THEORY PREDICTS ZERO FOR $T > T_c$

AFTER A BALLISTIC EXPANSION $\vec{r} = \frac{\hbar t}{m} \vec{q}$

THIS CORRESPONDS TO $\langle \delta n_\uparrow(\vec{r}) \, \delta n_\downarrow(-\vec{r}) \rangle$

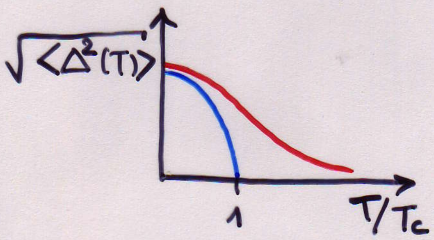
(SPIN CORRELATIONS AT OPPOSITE POINTS OF THE TRAP)

* FOR A SMALL TRAPPED SYSTEM

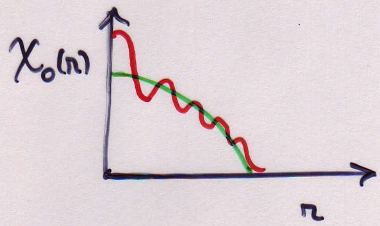
$$\langle \delta n_\uparrow(\vec{r}) \, \delta n_\downarrow(-\vec{r}) \rangle \propto |\chi_0(r)|^2$$

SUMMARY

FOR SMALL FERMI GASES CLOSE TO T_c
THE MEAN-FIELD BCS THEORY FAILS



THE BCS TRANSITION
IS SPREAD OUT



FLUCTUATIONS IN THE PAIRING
AMPLITUDE GIVE RISE TO "ANOMALOUS"
CORRELATIONS EVEN ABOVE T_c

→ SPIN-RESOLVED DENSITY
CORRELATION FUNCTION

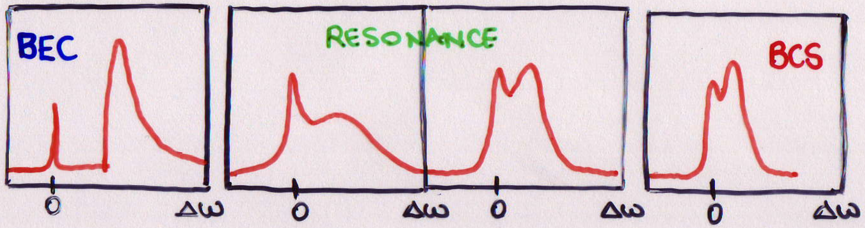
THIS IS A SIGNAL OF THE ONSET
OF COOPER PAIRING

PERSPECTIVES

* PAIRING FLUCTUATIONS IN THE BEC-BCS CROSSOVER

→ RECENT OBSERVATION OF THE PAIRING GAP IN RF SPECTRA

$T \simeq 0.5\,T_F$



Chin et al. Science **305** 1128 (2004)

* P-WAVE FESHBACH RESONANCES MEASURED FOR ^6Li

C. Salomon group at ENS

→ P-WAVE SUPERFLUIDITY WITH TUNABLE COUPLING STRENGTH

PAIRING FLUCTUATIONS IN TRAPPED FERMI GASES

SCUOLA NORMALE SUPERIORE
PISA , ITALY

ANNA MINGUZZI

LUCIANO VIVERIT
GEORG BRUUN
ROSARIO FAZIO