Vacuum energy:
a condensed matter primer

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Introduction

Λ vs vacuum energy, puzzles of dark energy

Quantum liquids

effective Quantum Field Theory and quantum vacuum;
equation of state for vacuum

Main cosmological problem:

Why is Λ so small?

Coincidence problem:

Why does Λ have its present value?

What is energy of false vacuum?

What happens with Λ at cosmological phase transition?

Static Universes:

Does Λ exist without gravity?

Einstein was right:

"grösster Fehler (Schnitzer)" is one of his most brilliant inventions

Cosmological Constant

*** Original Einstein equation with cosmological constant**

$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \Lambda g_{\mu\nu} = T^{\text{Matter}}_{\mu\nu}$$

gravitational field **matter as source of gravity**

*** Cosmological constant as vacuum energy (**Λ *moved to rhs***)**

$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = \Lambda g_{\mu\nu} + T^{\text{Matter}}_{\mu\nu}$$

gravitational field **vacuum energy as source of gravity** **matter as source of gravity**

*** Energy-momentum tensor for vacuum medium**

$$T^{\text{Vacuum}}_{\mu\nu} = \Lambda g_{\mu\nu}$$

$$\varepsilon_{vac} = T^{vac}_{00} = \Lambda \qquad P_{vac} = T^{vac}_{11} = T^{vac}_{22} = T^{vac}_{33} = -\Lambda$$

*** Equation of state for vacuum medium in relativistic QFT**

$$\boxed{P_{vac} = -\varepsilon_{vac}}$$

This equation of state is valid for any vacuum (even non-relativistic) since it follows from thermodynamics

Estimated cosmological constant

$$L = \frac{1}{16\pi G}\sqrt{-g}\,R + \sqrt{-g}\,\Lambda$$

Curvature term

Cosmological constant Λ

= vacuum energy density ε_{vac}

common generally accepted view:

vacuum energy ε_{vac} =
= zero-point energy of bosonic quantum fields +
+ negative energy of Dirac vacuum of fermions

Estimated vacuum energy

from zero-point energy of quantum fields

$$\Lambda_{theor} = (1/2)\sum_{\mathbf{bosons}} E(p) - \sum_{\mathbf{fermions}} E(p) \approx \pm c p^4_{\mathbf{SUSY}} + (\nu_{\mathbf{b}} - \nu_{\mathbf{f}}) c p^4_{\mathbf{Planck}}$$

zero-point energy

energy of Dirac vacuum

number of bosonic fields

number of Dirac fermions

$$\Lambda_{theor} = 10^{120} \Lambda_{upper\ limit}$$

in case of supersymmetry (SUSY)

$$\nu_{\mathbf{b}} = \nu_{\mathbf{f}}$$

$$\Lambda_{theor} = 10^{60} \Lambda_{upper\ limit}$$

Many-body systems
(quantum liquids and solids)

vacuum = ground state

matter = quasiparticles

elementary particles & quanta of fields — excitations above the ground state

superfluid ^4He	**Universe**	**superfluid ^3He-A**
ground state + bosonic qp	**vacuum + matter**	**ground state + fermionic qp**

bosonic quasiparticles:
phonons -- quanta of sound waves

$$E(p)=cp$$

c -- speed of sound

fermionic quasiparticles
with anispotropic linear spectrum

$$E^2(\mathbf{p})=c_x^2p_x^2 + c_y^2p_y^2 + c_z^2p_z^2$$

Estimated vacuum energy

from zero-point energy of quantum fields

$$\Lambda_{\rm theor} = (1/2) \sum_{\rm phonons} E(p)$$

$$\sqrt{-g}\, E^4_{\rm Planck}$$

$$\Lambda_{\rm theor} = - \sum_{\rm fermions} E(\mathbf{p})$$

$$- \sqrt{-g}\, E^4_{\rm Planck}$$

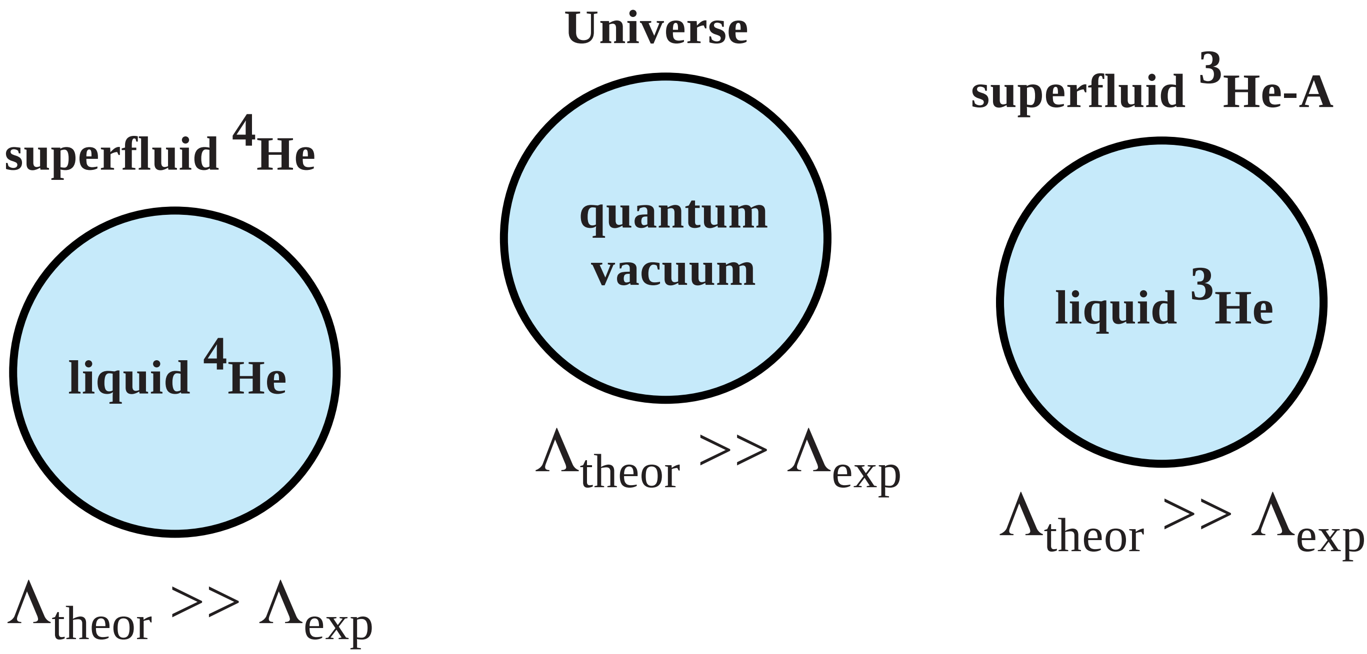
$E_{\rm Planck}$ - effective Planck energy scale of order Debye temperature

$$\sqrt{-g} = c^{-3}$$

$$\sqrt{-g} = 1/c_x c_y c_z$$

$\Lambda_{\rm exp}$ is many orders of magnitude smaller in both liquids

Quantum vacua of relativistic QFT & in quantum liquids have the same vacuum energy problem as in our Universe



What is wrong with our theoretical estimations?

In quantum liquids we know the underlying microscopic (trans-Planckian) physics

This allows us to find the answer

**The answer is so general,
that it must be applicable to any QFT**

Theory of Everything
in quantum liquids

liquid ^{4}He

10^{23} atoms

liquid ^{3}He

10^{23} atoms

Microscopic theory

Many-body
Schrödinger quantum mechanics
for N atoms

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \sum_{i=1}^{N}\sum_{j=i+1}^{N} V(\mathbf{r}_i - \mathbf{r}_j)$$

Quantum Field Theory in quantum liquids

Abrikosov, Gor'kov & Dzyaloshinskii
Quantum Field Theoretical Methods in Statistical Physics

QFT from second quantization

thermodynamic potential relevant for QFT:

$$\hat{H}_{QFT} = \hat{H} - \mu\hat{N} = \int d^3x\, \psi^+\left(-\frac{\Delta}{2m} - \mu\right)\psi \;+$$
$$+ \int d^3x d^3y V(x-y)\, \psi^+(x)\psi^+(y)\psi(y)\psi(x)$$

operator ψ
bosonic
in **^{4}He,**
fermionic
in **^{3}He**

vacuum energy relevant for QFT:

$$E_{QFT} = E - \mu N = \langle\hat{H}\rangle_{vac} - \mu\langle\hat{N}\rangle_{vac}$$

Gibbs-Duhem relation for quantum vacuum

$$E_{vac} = E - \mu N = <\hat{H}>_{vac} - \mu<\hat{N}>_{vac}$$

Thermodynamic Gibbs-Duhem relation for equilibrium state

$$E - \mu N - TS = - P$$

Gibbs-Duhem identity for equilibrium vacuum (T=0)

$$E_{vac} = \varepsilon_{vac} V = E - \mu N = - PV$$

P **is vacuum pressure**

Vacuum energy

$$E_{vac} = E - \mu N$$

does not depend on choice of zero !

$$\varepsilon_{vac} = - P_{vac}$$

This equation reproduces the equation of state for vacuum in relativistic QFT and corresponds to the cosmological constant

$$\Lambda = \varepsilon_{vac} = - P$$

Cosmological constant problem #1
Why Λ is not big

Vacuum energy of equilibrium quantum liquids

liquid ^4He

liquid ^3He

Contribution of zero-point motion of quantum fields

$$\Lambda_{theor} = (1/2) \sum_{phonons} cp$$

$$\sqrt{-g}\, E^4_{Planck}$$

$$\Lambda_{theor} = - \sum_{fermions} cp$$

$$- \sqrt{-g}\, E^4_{Planck}$$

Exact result from Gibbs-Duhem relation applied to vacuum

$$P = -(E - \mu N)/V = -\varepsilon_{vac}$$

- P: vacuum pressure
- μ: chemical potential
- N: particle number
- ε_{vac}: vacuum energy density

For droplet **isolated** from environment the external pressure is zero:

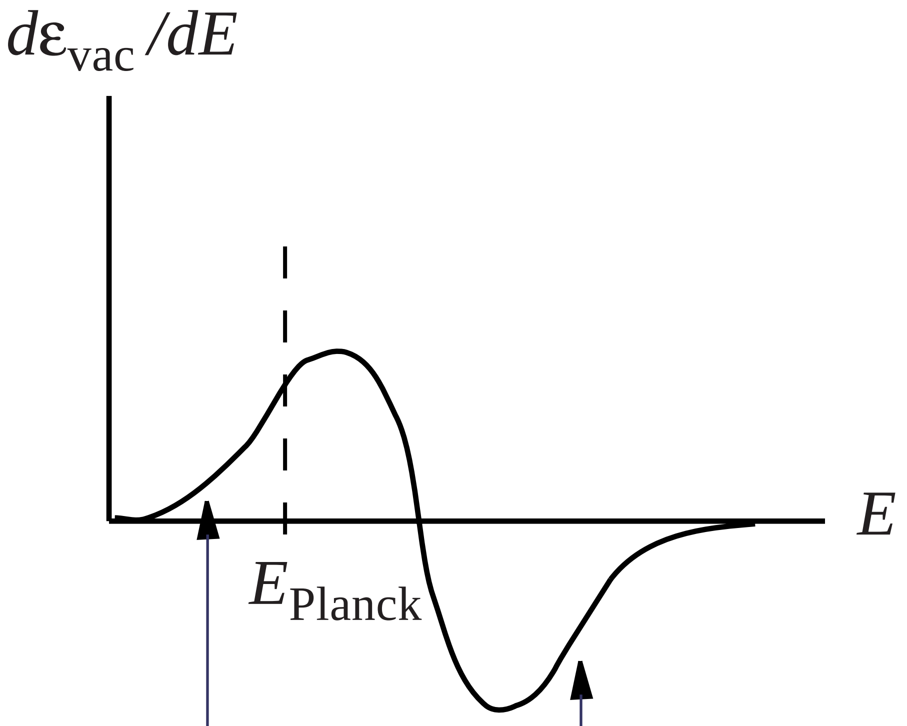
$$\varepsilon_{vac} = - P = 0$$

$$\Lambda_{exact} = \varepsilon_{vac} = 0$$

Lesson from quantum liquids

if vacuum is in thermodynamic equilibrium, microscopic degrees of freedom (atomic or trans-Planckian) exactly compensate (sub-Planckian) contribution of zero-point motion of QFT

Spectrum of vacuum energy



contribution of zero-point fluctuations:

$$d\varepsilon_{vac}/dE = (1/2)N(E)E \sim E^{3}$$

$$\varepsilon_{vac} \sim \int_0^{E_{Planck}} dE\, E^{3} = E_{Planck}^{4}$$

contribution of microscopic degrees of freedom

$$\varepsilon_{vac} = \int_0^{\infty} dE\, (d\varepsilon_{vac}/dE) = 0$$

Cosmological constant problem #2

Why is cosmological constant nonzero?

Vacuum energy is proportional to perturbations of vacuum state

Matter as a source of perturbation of the vacuum:

liquid ^{4}He
$T \neq 0$

Universe without gravity
$T \neq 0$

liquid ^{3}He
$T \neq 0$

matter:

radiation, ultrarelativistic particles or `relativistic' quasiparticles

with equation of state

$$\varepsilon_{matter} = 3P_{matter} = \gamma T^4 \sqrt{-g}$$

bosonic (quasi)particles

$$\gamma = \pi^2/30$$

fermionic (quasi)particles

$$\gamma = 7\pi^2 N_F/120$$

Equation of state for vacuum

$$\varepsilon_{vac} = - P_{vac}$$

Equilibrium condition for isolated liquid or for Universe

$$P = P_{vac} + P_{matter} = 0$$

$$\Downarrow$$

$$\varepsilon_{vac} = (1/3)\varepsilon_{matter}$$

This simulates the response of vacuum to matter in the Universe obeying special relativity where the Newton constant $G=0$

Vacuum energy density ε_{vac} in different Universes

with matter obeying general equation of state

$$P_{matter} = w\varepsilon_{matter}$$

before we used $w=1/3$

static Universe without gravity

$$\varepsilon_{vac} = w\varepsilon_{matter}$$

quantum liquid

static Einsten closed Universes

$$\varepsilon_{vac} = (1/2)(1+3w)\varepsilon_{matter}$$

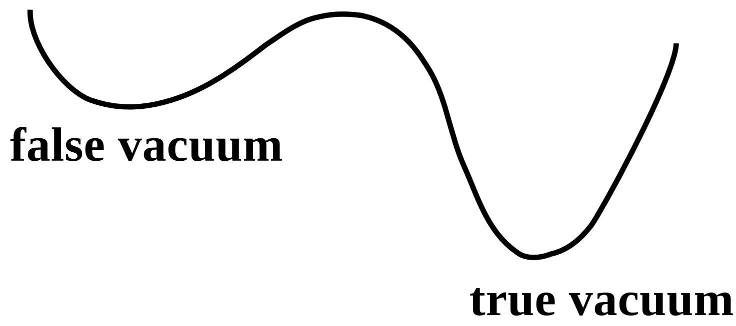
our accelerating Universe

$$\varepsilon_{vac} = 2 \div 3\ \varepsilon_{matter}$$

our Universe is not very far from equilibrium

Cosmological constant problem #3

What is cosmological constant in false vacuum?



Gibbs-Duhem relation is applicable to any equilibrium vacuum

equilibrium false vacuum

$$\varepsilon_{vac} = 0$$

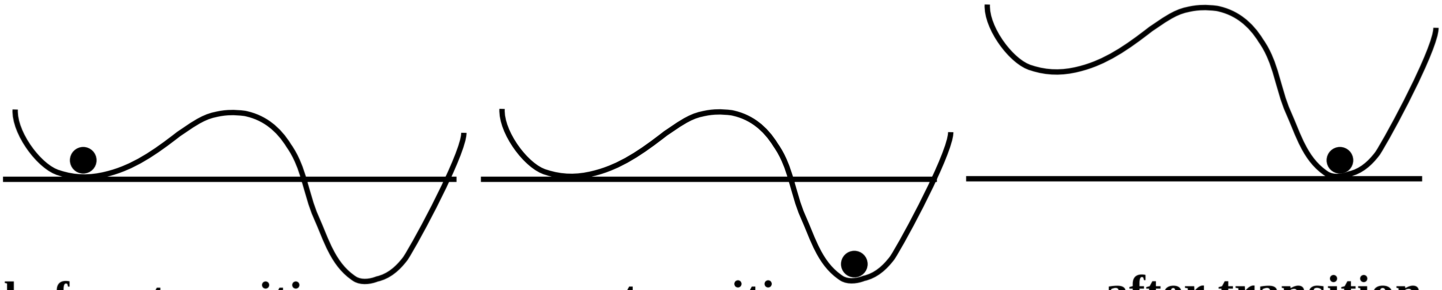
$\Lambda = 0$ **in both of them**

equilibrium true vacuum

How and why does the phase transition occur ???

Cosmological constant problem #4

What happens after phase transition?



before transition

equilibrium
false
vacuum

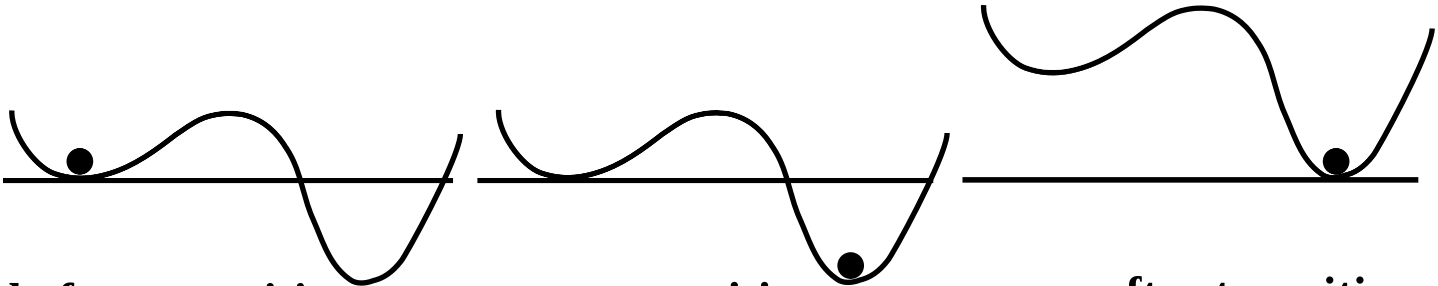
transition

non-equilibrium
true
vacuum

after transition

microscopic degrees of freedom readjust themselves to new equilibrium vacuum

Example of A to B transition in superfluid 3He



before transition

equilibrium
A phase

$\varepsilon_{false} = \varepsilon_A - \mu_A n_A$

transition

non-equilibrium
B phase

$\varepsilon_{true} = \varepsilon_B - \mu_A n_B$

after transition

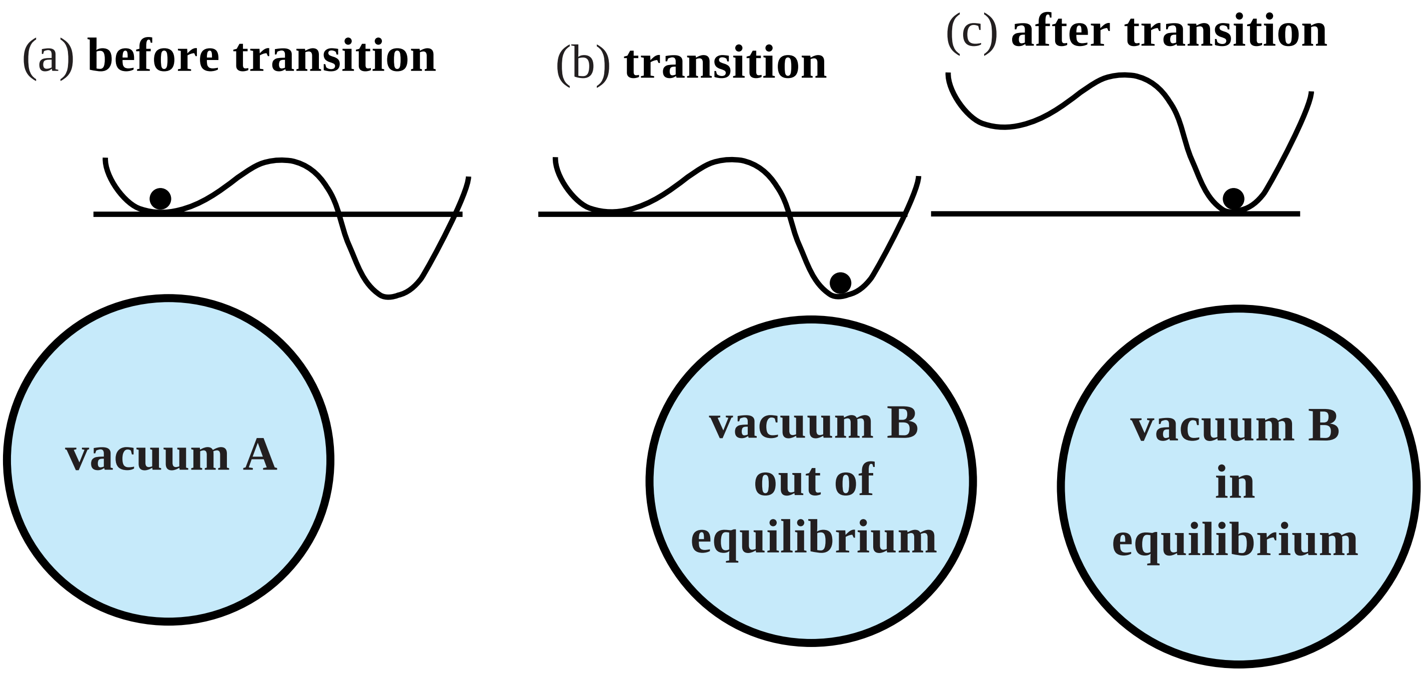
equilibrium
B phase

$\varepsilon_{true} = \varepsilon_B - \mu_B n_B$

microscopic degrees of freedom which readjust themselves:
chemical potential μ **or density of atoms** n
transition occurs at fixed $\mu = \mu_A$ **,**
after that chemical potential shifted to $\mu = \mu_B$

Cosmological constant problems #5

How big is the vacuum response to match the new equilibrium after transition?



microscopic parameters adjust themselves to new equilibrium

vacuum response to absorb electroweak energy after transition:

$$\frac{\delta E_{\mathrm{Planck}}}{E_{\mathrm{Planck}}} \sim \frac{E^4_{\mathrm{electroweak}}}{E^4_{\mathrm{Planck}}}$$

vacuum response to absorb AB transition in 3He:

$$\frac{\mu_A - \mu_B}{\mu_A} \sim \frac{T_c^2}{E_F^2}$$

response of the vacuum is negligible because of its huge zero-point energy, but this energy is not gravitating

Cosmological constant problem # N

The main remaining cosmological *constant* problems:

Dynamics of Λ

How does Λ adjust itself to dynamics of evolving Universe ?

How does Λ relax after cosmological phase transition ?

. . .

Dynamics of Λ requires modification of Einstein equations

Conclusion

The problem of vacuum energy is more fundamental than the problem of Λ
It is common for relativistic & non-relativistic systems, with or without gravity

It requires the general thermodynamic analysis applicable to any system

Gibbs-Duhem relation is the proper tool to study vacuum close to equilibrium

Thermodynamics of quantum condensed-matter systems shows how cancellation of vacuum energy occurs without fine-tuning:

microscopic degrees if freedom compensate zero-point energy of quantum fields

Thermodynamics allows to obtain static response of the vacuum to: matter, gravity, phase transition, etc.

Dynamics of Λ requires introduction of dissipation into Einstein equations, this can be done using schemes developed in quantum liquids and superconductors

references:
gr-qc/0405012 , hep-ph/0309144, gr-qc/0304103, gr-qc/0304061