From the Cosmological Constant Problem to Dark Energy

ROLAND TRIAY

Université de Provence
and Centre de Physique Théorique CNRS (UMR 6207)
*Luminy Case 907, F 13288 Marseille Cedex 9, France**

Outline

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Status of the cosmological constant

The reason why Λ was wrongly understood as a free parameter in Einstein's equations has an historical origin, known as "Einstein's bigest blunder": he used it for describing the data according to a static cosmological solution (Einstein 1917) when (later on) a general expansion of the universe was observed (Hubble 1927). Such an attitude became an authority argument in favour of $\Lambda = 0$, until acceleration of the cosmological expansion was established.

The status of the cosmological constant Λ has long been discussed (Souriau 1977; Felten & Isaacman 1986; Charlton & Turner 1987; Sandage 1988; Carroll, Press, Turner 1992), while it is clearly established (Souriau 1964) in General Relativity (GR) as *universal constant*, as similalrly as the one of Newton constant. It intervenes at large scale for describing the gravitational field. If $\Lambda \neq 0$ then the dynamics of the cosmological expansion can be investigated by rescaling Einstein equations with adapted units (Souriau & R. Triay 1997).

Observational status of the cosmological constant

Probably because of some *a priori* in mind, estimates such as

$$\Lambda < 2\,10^{-55}\,\mathrm{cm}^{-2}$$

from dynamics of galaxies in clusters (Peach 1970) or

$$-2\,10^{-56}\,\mathrm{cm}^{-2} \leq \Lambda < 4\,10^{-56}\,\mathrm{cm}^{-2}$$

from the minimum age of the universe and the existence of high redshift objects (Carroll, Press, Turner 1992), were interpreted (for arguing) in favor of a vanishing value.

On the other hand, subsequent estimates based on the redshift distance relation for brightest cluster galaxies (Bigot, Fliche, Triay 1988, Bigot & Triay 1989) and for quasars (Fliche & Souriau 1979; Fliche, Souriau & Triay 1982; Bigot, Fliche, Triay 1988; Triay 1989) provided us unambigously with a non zero cosmological constant

$$\Lambda \sim 3h^2\,10^{-56}\,\mathrm{cm}^{-2}, \qquad h = H_\circ/100\,km\,s^{-1}\,Mpc^{-1}$$

Nowadays, it is generally believed that

$$\Lambda \sim 2h^2\,10^{-56}\,\mathrm{cm}^{-2}$$

is required for accounting of observations from SN (J.V. Perlmutter *et al* 1998,1999; Schmidt *et al.* 1998; Riess *et al.* 1998,2004) and CMB (Sievers *et al.* 2002, Netterfield *et al.* 2002, Spergel *et al.* 2003, Benoit etal *et al.* 2003)

Principle of General Relativity applied to gravity

The gravitational field and the gravitational sources are characterized respectively by the metric tensor $g_{\mu\nu}$ on the space-time manifold V_4 and by a *vanishing divergence* stress-energy tensor $T_{\mu\nu}$.

The gravitational field equations satisfy the

Principle of general relativity (Souriau 1964)

i.e., they must be invariant with respect to the action of diffeomorphism group of V_4. Their most general form reads in term invariants and can be writen as follows

$$T_{\mu\nu} = -A_0 F^{(0)}_{\mu\nu} + A_1 F^{(1)}_{\mu\nu} + A_2 F^{(2)}_{\mu\nu} + \dots$$

where

$$F^{(0)}_{\mu\nu} = g_{\mu\nu}, \quad F^{(1)}_{\mu\nu} = S_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

are the invariants of order 0 and 2 respectively. $R_{\mu\nu}$ stands for the Ricci tensor and R the scalar curvature, $F^{(n)}_{\mu\nu}$ is a function which reads in term of invariants of order $2n$, and finally the A_n are *coupling constants*. The invariants of order ≤ 2 are uniquely defined (*i.e.* $F^{(n)}_{\mu\nu}$ with $n \leq 1$), but the functions $F^{n}_{\mu\nu}$ with $n \geq 2$ have to be derived from additional principles or suggested by observations.

Dimensional analysis of gravitational field equations

The dimensional analysis of field equations unables us to estimate the relative contributions of constants A_n for describing the gravitational field.

According to GR, time can be measured in unit of length

$$1\mathrm{s} = 2.999\,792\,458\,10^{10}\,\mathrm{cm}$$

i.e. the speed of the light $c = 1$[a]. Let us choose units of mass M and of length L (only two fundamental units can be chosen, the third one is derived).

The correct dimensional analysis of GR sets the covariant metric tensor to have the dimension

$$[g_{\mu\nu}] = L^2$$

and thus one has

$$[g^{\mu\nu}] = L^{-2}, \quad [R_{\mu\nu}] = 1, \quad [R] = L^{-2}$$

Since the specific mass density and the pressure belong to the mixed tensor T^μ_ν one has

$$[T_{\mu\nu}] = ML^{-1}$$

Therefore, the identification of units of constants

$$[A_0] = ML^{-3}, \quad [A_1] = ML^{-1}, \quad \ldots \tag{1}$$

shows that the larger their order n the smaller their effective scale. Namely, the contribution of A_0 dominates at large scale, the one of A_1 at smaller scale, and so on...

[a]This means that any statement which mentions c varying is meaningless in GR

Identification of universal constants

In order to identify these constants, we use Newton approximation of field equations, with Poisson equation

$$\mathrm{div}\vec{\mathrm{g}} = -4\pi \mathrm{G}\rho + \Lambda$$

where $\vec{g}$ stands for the gravitationnal acceleration field due to sources defined by a spectific density ρ. The identification gives then

$$G = \frac{1}{8\pi A_1} = 7.4243 \times 10^{-29}\ \mathrm{cm\,g^{-1}}, \qquad \Lambda = \frac{A_0}{A_1} \approx 2\,h^2 \times 10^{-56}\ \mathrm{cm}^{-2}$$

The related acceleration reads

$$\vec{g} = \left(-G\frac{M}{r^3} + \frac{\Lambda}{3}\right)\vec{r}$$

If $\Lambda > 0$ then the gravitational force around a mass M is attractive at distance $r < r_\circ = \sqrt[3]{3MG/\Lambda}$ but repulsive at $r > r_\circ$,

Hence, one understands that the Λ term has the status of universal constant as Newton gravitational constant G. In other words, "any statement which enables us to assume $\Lambda = 0$ within GR, could be used as well as for Newton constant of gravitation G".

Because of their respective dimensions

$$[G] = LM^{-1}, \qquad [\Lambda] = L^{-2}$$

the contribution of Λ for describing the gravitational field is expected at much larger scale than the one of G, the transition scale is of order of $1/\sqrt{\Lambda}$.

Solving Problems of Standard Cosmology

- The Age Problem (FST 1982, Souriau & Triay 1997)
- The Flatness Problem (Triay 1997)
  Actually Dicke's coincidence problem 1970

$$1 = \Omega_\Lambda + \Omega_k + \Omega_m + \Omega_\gamma$$

$$\Omega_k < -1$$

- The origin of Inertia (Mach's Principle)
- Electric neutrality & Antimatter Problems (Fliche, Souriau & Triay 1982)

Modeling gravitational structures

The space-time is constrained by the presence of gravitational sources as described by means of tensor $T_{\mu\nu}$. Each right hand terms contributes within its effective scale for describing the geometry. This is the reason why gravitational structures within scales of order of solar system can be described by limiting the expansion solely to Einstein tensor $S_{\mu\nu}$, when the first term is also required in Cosmology. If $\Lambda \neq 0$ then one can units so that field equations can be written in a normalized form (i.e., $A_0 = A_1 = 1$) as follows

$$T_{\mu\nu} = -g_{\mu\nu} + S_{\mu\nu} + A_2 F^{(2)}_{\mu\nu} + \dots \tag{2}$$

Such a choice, herein called "*gravitational units*", which sets the units of time $1/\sqrt{\Lambda}$ and of mass $1/(8\pi G\sqrt{\Lambda})$, is adapted for describing gravitational structures up to largest scales, and in particular for the dynamics of the cosmological expansion.

The Cosmological constant Problem

The investigation of the contribution to the gravitational field of the vacuum energy density due to quantum fluctuations leads to the so called "cosmological constant problem" (Carroll 2001, Padmanabhan 2003).

Namely, the predicted value from quantum fields theories and the one measured from observations at cosmological scale differs by 120 orders of magnitude.

Other estimations of this quantum effect from the point of view of the standard Casimir energy calculation scheme (Zeldovich 1967) provide us with discrepancies of ~ 37 orders of magnitude (Cherechnikov 2002).

Such a result has a clear explanation by using gravitational units, since the Planck constant is equal to

$$\hbar \sim 10^{-120}$$

which shows that such a very small value for the *quantum action unit* (which has to be compared to $\hbar = 1$ when quantum units are used instead) shows clearly that Einstein equations (*i.e.* field equations with $n \leq 1$) are not adapted for describing quantum physics (Triay 2002,2004).

Modeling the cosmological expansion as quantum structure

Let us assume a quantum approach

$$T_{\mu\nu}^{vac} = \rho_{vac}\, g_{\mu\nu}, \qquad \rho_{vac} = \hbar k_{\max}$$

with

$$\rho_{vac}^{EW} \sim 2\,10^{-4}\ \mathrm{g\,cm^{-3}}, \qquad \rho_{vac}^{QCD} \sim 1.6\,10^{15}\ \mathrm{g\,cm^{-3}}, \qquad \rho_{vac}^{Planck} \sim 2\,10^{89}\ \mathrm{g\,cm^{-3}}$$

the dynamics of the cosmological expansion satisfies the field equations

$$T_{\mu\nu}^{mat} + T_{\mu\nu}^{rad} + T_{\mu\nu}^{vac} = -g_{\mu\nu} + S_{\mu\nu} + A_2 F_{\mu\nu}^{(2)} + \ldots$$

Discussion

- the contribution of matter $\rho_{mat} \sim 0.3h^2 \times 1.8783\,10^{-29}\ \mathrm{g\,cm^{-3}}$
- hypotheses on the spatial distribution of sources (void between sources)
- the effective scale is not quantum

Suggestion

- $\rho_{vac} \propto F_{\mu\nu}^{(2)}$
- Quantum Gravity

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