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INHOMOGENEOUS IMPURITY DISTRIBUTION IN FERMI SUPERFLUIDS

Erkki Thuneberg
Department of Physical Sciences, University of Oulu

E.T, S.K. Yip, M. Fogelström, and J.A. Sauls, Phys. Rev. Lett. **80**, 2861 (1998).
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http://boojum.hut.fi/research/theory/aerogel.html

Content

Impurities in Fermi superfluids

- quasiclassical theory
- impurity averaging

Application to superfluid ^{3}He in aerogel

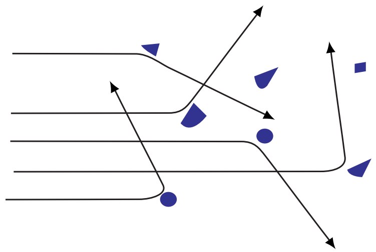
- homogeneous distribution of impurities
- a model of inhomogeneous impurity distribution
- discussion

Impurities in metals

Parameters:
- scattering cross section σ
- density of impurities n
- mean free path for quasiparticles

$$\frac{1}{\ell} = n\sigma$$

Scattering causes electrical resistance in normal metals



Superconductors (usual case)

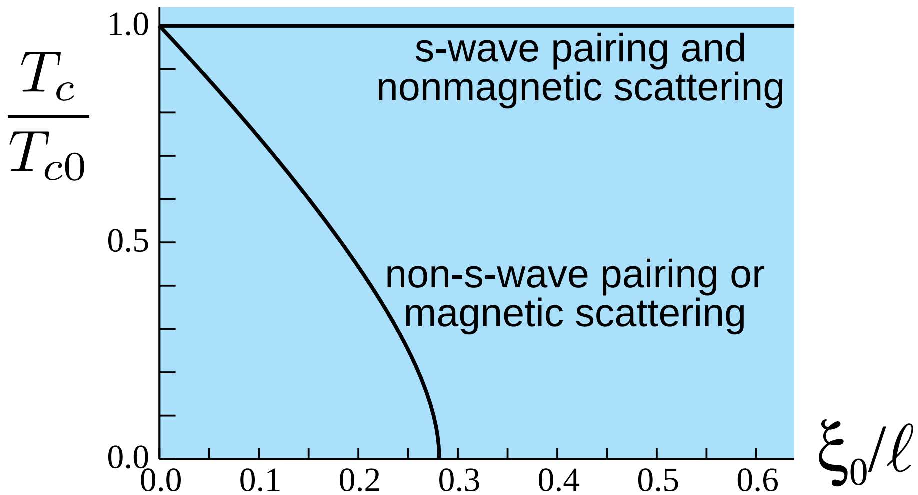
- T_c not changed (Anderson theorem)
- Ginzburg-Landau parameter increases

but

- magnetic impurities in usual superconductors, or

- usual impurities in unconventional superconductors (p, d, ... wave pairing)

$\Rightarrow$



(Abrikosov-Gorkov 1961)

Quasiclassical scattering theory

parameters:

- Fermi wave length λ_F
- coherence length

$$\xi_0 = \frac{\hbar v_F}{2\pi k_B T_c}$$

- scattering cross section σ
- density of impurities n
- $n\sigma = 1/\ell$

Assumption:

$$\lambda_F, \sqrt{\sigma} \ll \xi_0, \ell \qquad (1)$$

Take leading terms, ignore terms that are smaller by factors λ_F/ξ_0, etc.

Technical tool: take an average over the locations of the impurities.

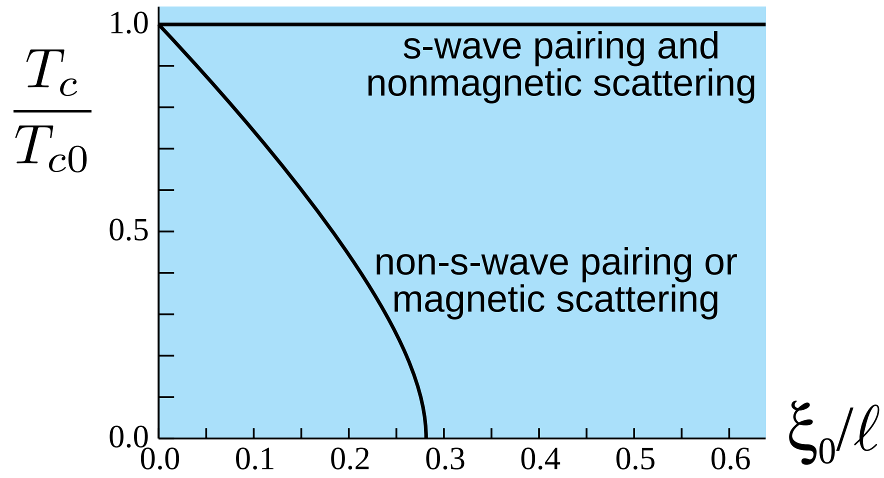
However, due to assumptions (1), this implies that all fluctuations in the impurity density are lost.

$\Rightarrow$

Instead of true discrete impurities there is a continuous scattering medium.

$\Rightarrow$

"Homogeneous scattering model" (HSM)



Quasiclassical equations

4x4 matrix Green's function $\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m)$:

$$[i\epsilon_m \widehat{\tau}_3 - \widehat{\nu} - \widehat{\Delta} - \widehat{\rho}, \widehat{g}] + i\hbar \mathbf{v}_{\mathsf{F}} \cdot \nabla_{\mathbf{r}} \widehat{g} = 0$$

$$\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m)\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m) = -\pi^2$$

$$\widehat{\rho}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m) = n(\mathbf{r})\widehat{t}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m).$$

Equation for the scattering matrix $\widehat{t}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}', \epsilon_m)$

$$\widehat{t}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}', \mathbf{r}, \epsilon_m) = \widehat{v}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}') + N(0)\langle \widehat{v}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}'')\widehat{g}(\widehat{\mathbf{k}}'', \mathbf{r}, \epsilon_m)\widehat{t}(\widehat{\mathbf{k}}'', \widehat{\mathbf{k}}', \mathbf{r}, \epsilon_m)\rangle_{\widehat{\mathbf{k}}''}.$$

Energy functional

$$\Omega = \int d^3r [\frac{1}{V_{BCS}} \langle |\Delta(\widehat{\mathbf{k}}, \mathbf{r})|^2 \rangle_{\widehat{\mathbf{k}}} + \frac{1}{2} N(0) T \sum_{\epsilon_m} \int_0^{\Delta} \frac{d\Delta}{\Delta} \langle \mathrm{Tr}_4 [\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m) \widehat{\Delta}(\widehat{\mathbf{k}}, \mathbf{r})] \rangle_{\widehat{\mathbf{k}}}]$$

$+$ terms arising from Fermi-liquid corrections $\widehat{\nu}$.

Need a better theory?

- in high T_c superconductors $\sqrt{\sigma}/\xi_0$ is not negligible.

$\Rightarrow$

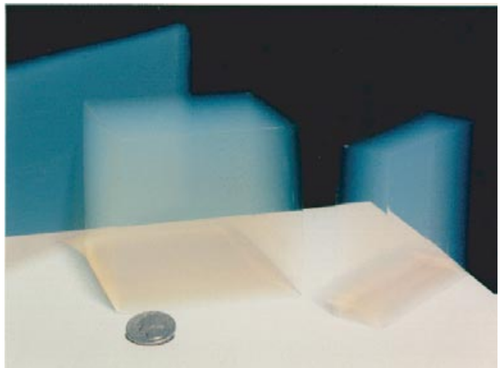
Fluctuations of the impurity density are important.
(Franz, Kallin, Berlinsky, and Salkola 1997)

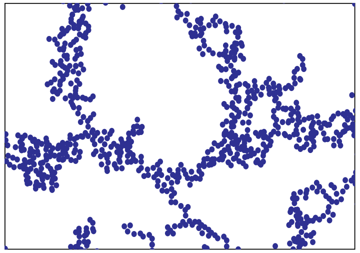
- superfluid ^{3}He in aerogel

Impure superfluid ^{3}He

^{3}He is a naturally pure substance

Impurity can be introduced by porous aerogel





- strands of SiO_2
- typically 98% empty
- small angle x-ray scattering $\Rightarrow$ homogeneous on a scale above ≈ 100 nm

Compare that to

$$\xi_0 = 16 \ldots 74 \text{ nm},$$

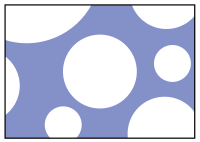
depending on pressure.

Experiments:
HSM has qualitative success, but insufficient quantitatively.

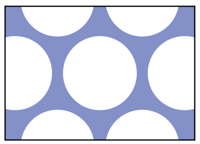
Models

Use quasiclassical theory with a location dependent impurity density $n(\mathbf{r})$.

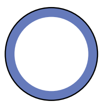
Homogeneous scattering model



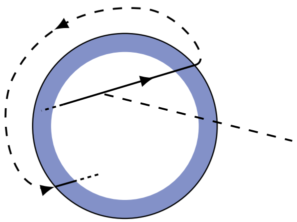
Random voids
- not simple enough to calculate



Periodic voids
- not simple enough to calculate



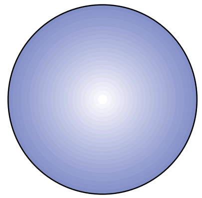
Isotropic inhomogeneous scattering
- spherical unit-cell approximation
- (quasi)periodic boundary condition



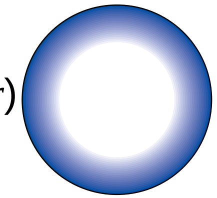
Isotropic inhomogeneous scattering model (IISM)

parameters:
- R, radius of the unit cell
- $n(r)$, scattering profile
- ℓ, average mean free path

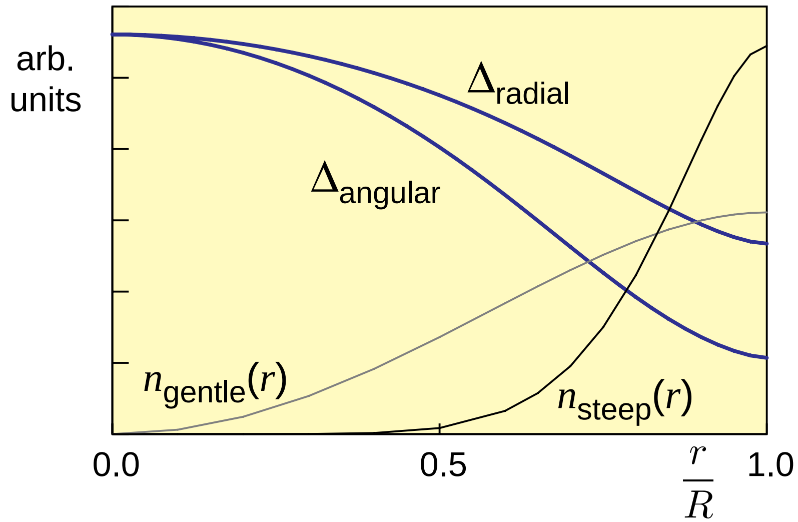
$n_{\text{gentle}}(r)$



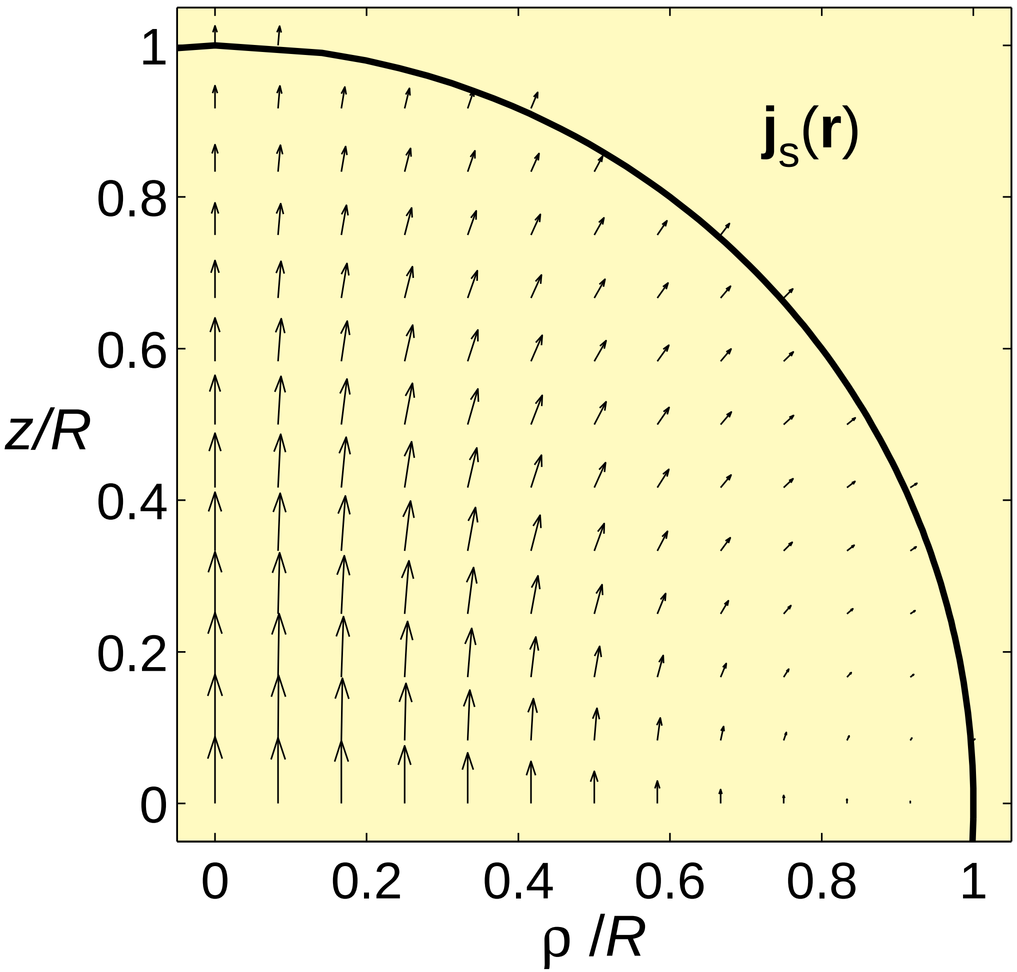
$n_{\text{steep}}(r)$



Order parameter in distorted B phase

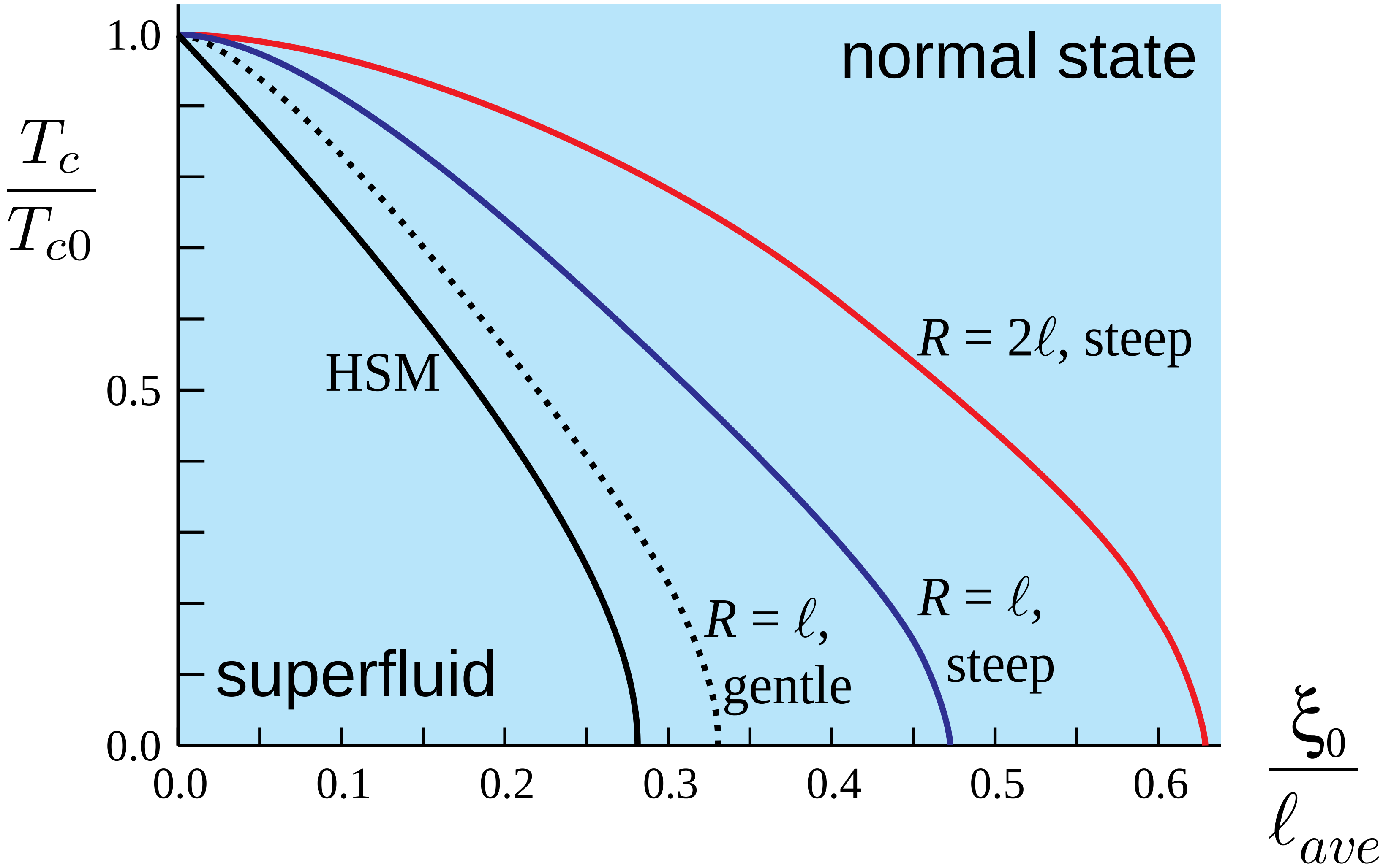


Supercurrent



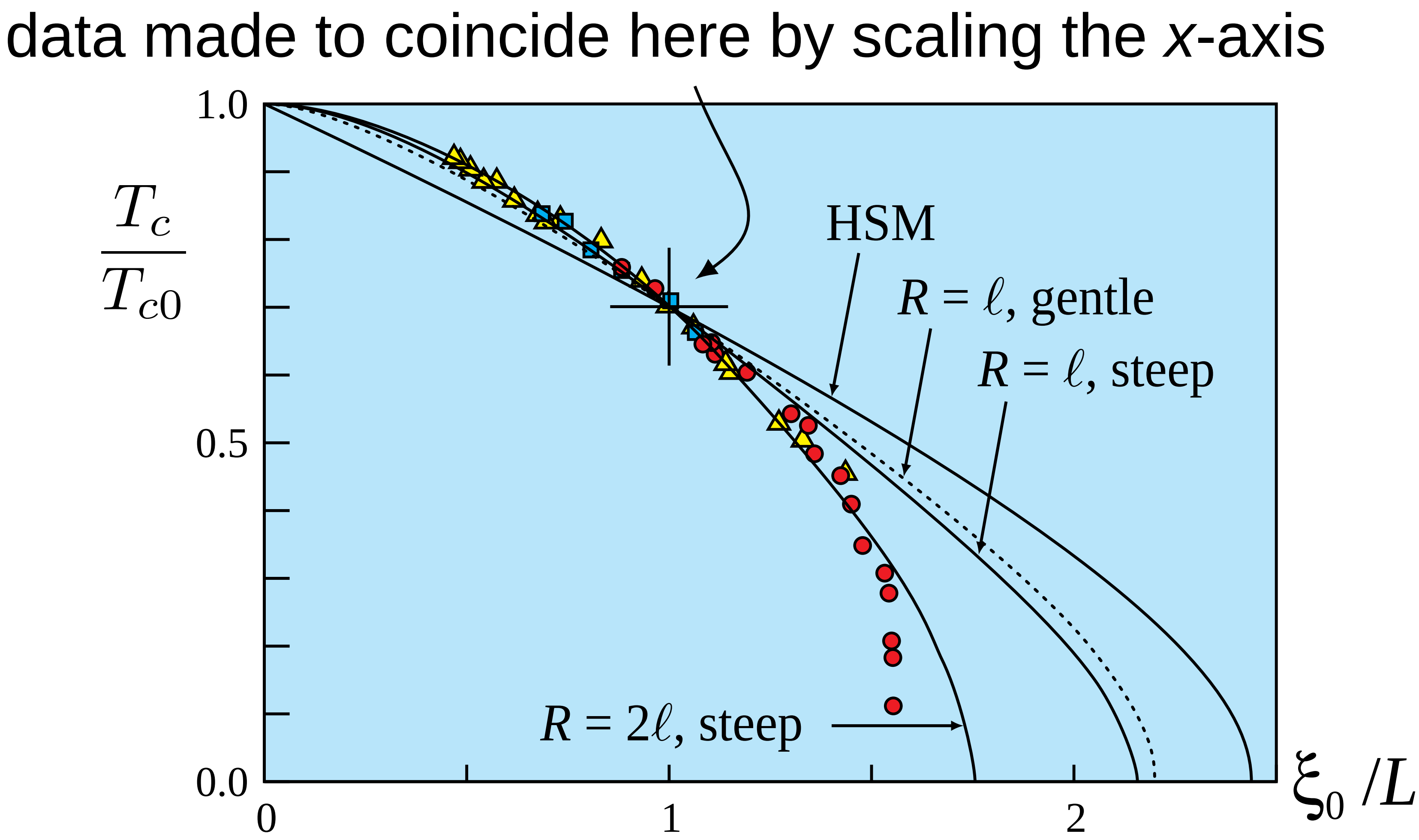
$\mathbf{j}_{\rm S}$ in cylindrical coordinates ρ, ϕ, z.

Transition temperature



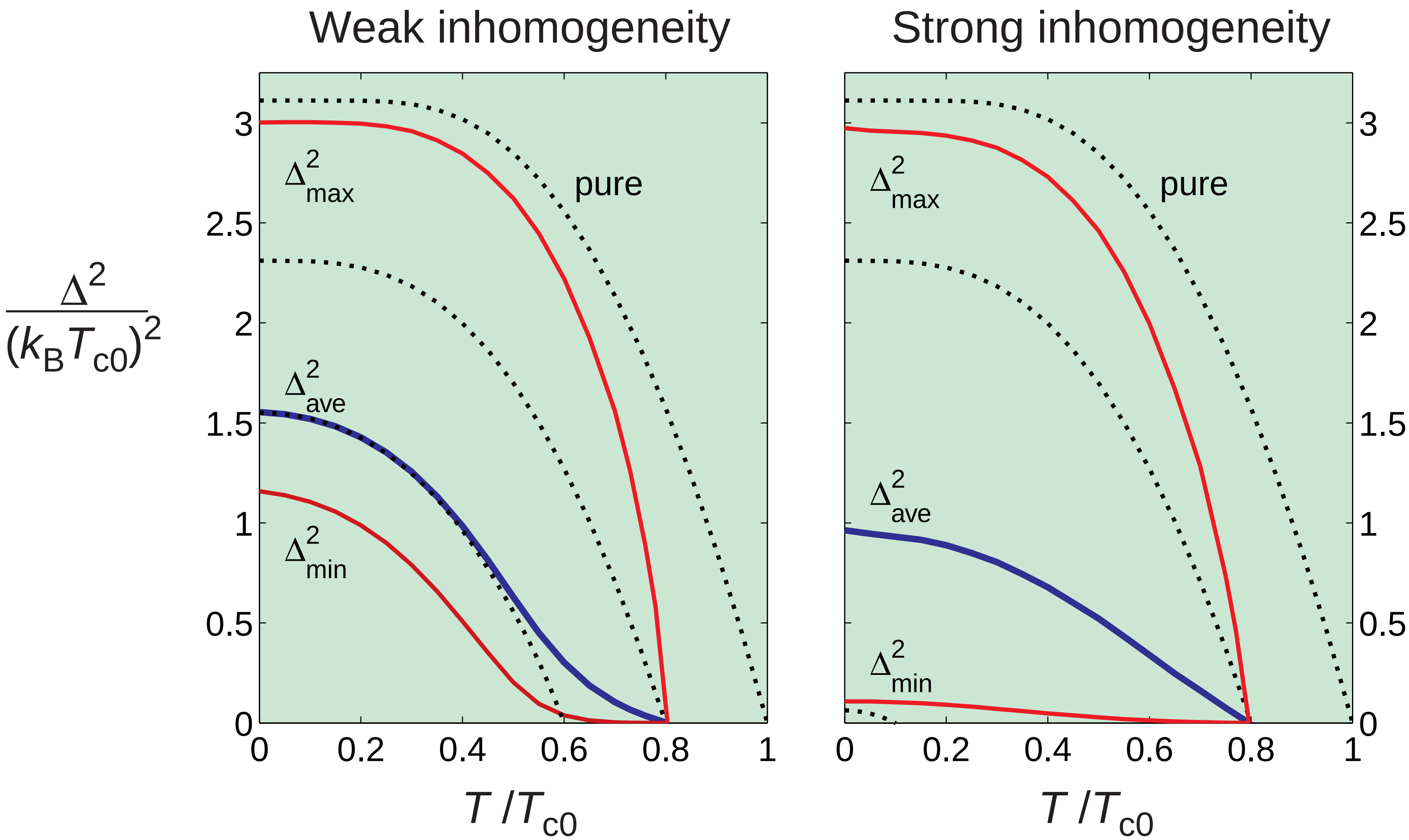
Transition temperature

Comparison to experiments (vary pressure)

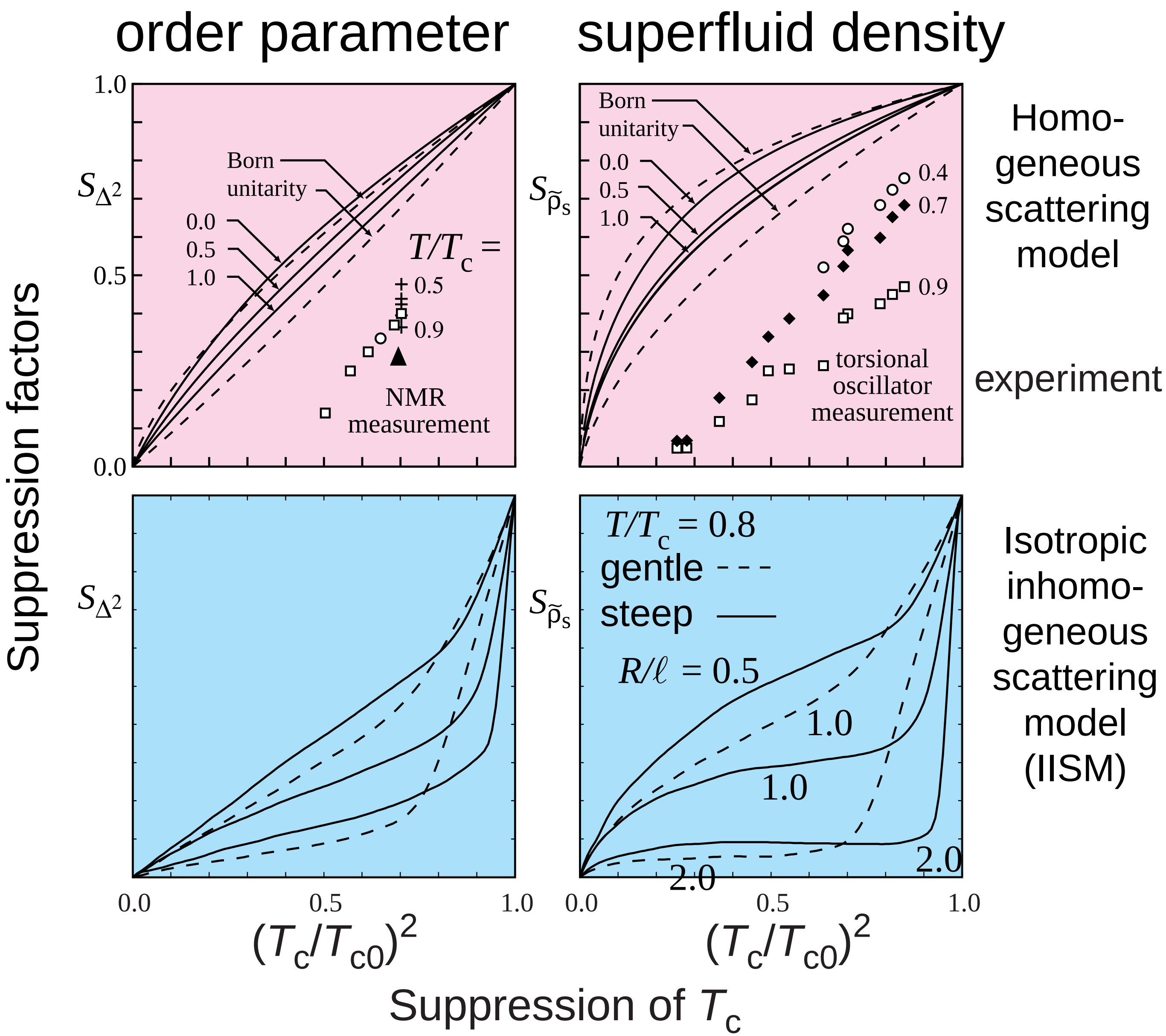


Porto et al (1995), Sprague et al (1995), Matsumoto et al (1997)

Temperature dependence of the order parameter



Suppression of



Comparison to experiments

The IISM gives better fit to experiment than HSM.

The optimal radius of the unit cell $R \approx 140$ nm is on the same order of magnitude than the aerogel correlation length ≈ 84 nm.

No perfect fit to experiments
- possible reason: IISM has only one length scale R.

Inhomogeneous order parameter

Inhomogeneity of order parameter $A_{\mu i}(\boldsymbol{r})$ essential
- constraining to constant $A_{\mu i}$ results in HSM

This inhomogeneity is neglected in determining the equilibrium A-B transition line (Baumgardner and Osheroff, 2004)

A-like phase

IISM: suppression of Δ nearly the same as for B phase
$\Rightarrow$ no exotic phase can compete with it.

Random variation in aerogel makes A-phase orientation to change randomly $\Rightarrow \langle A_{\mu i}(\boldsymbol{r})\rangle =$ small
Fomin: "A phase is not robust"

However, all physical properties depends on averaged square, e.g.

$$\langle F_{\mathrm{D}}\rangle = g_{\mathrm{D}}\langle A^*_{ii}A_{jj} + A^*_{ij}A_{ji} - \tfrac{2}{3}A^*_{\mu i}A_{\mu i}\rangle. \qquad (2)$$

$\Rightarrow$ All measurable properties A-phase like

Cooling from normal state

Assume aerogel in absence of bulk liquid

The transition always takes place first to the polar phase $A_{\mu i} = \Delta(\boldsymbol{r})\hat{d}_\mu \hat{m}_i$. This state is localized to a favorable region.

At some lower temperature a second component develops $A_{\mu i} = \hat{d}_\mu[\Delta_1(\boldsymbol{r})\hat{m}_i + \Delta_2(\boldsymbol{r})\hat{n}_i]$. A-phase is favored over planar phase by strong-coupling effects.

This may take place independently on many locations.

These regions grow together with decreasing T and form a disordered A phase.

Only in very exceptional (isotropic) locations B phase is nucleated instead of A.

Conclusions

Quasiclassical theory: inhomogeneous scattering modelled by $n(\boldsymbol{r})$.

Isotropic inhomogeneous scattering model (IISM):

- the simplest model of inhomogeneous scattering that reduces to homogeneous medium on a large scale

- computationally much heavier than HSM

- ^{3}He in aerogel: IISM clearly better than HSM, but still not perfect

- calculation of other properties? (specific heat, vortex states ...)

- application to other superfluids?

Links: http://boojum.hut.fi/research/theory/aerogel.html